

For questions such as this, we have to read carefully and consider the context in which the underlined word (in this case, “marked”) appears in order to settle on the best definition for this word among the answer choices.

The passage informs us that Mrs. Adams really likes vases and that her husband gave her a vase each year as a present. The passage also informs us that Mrs. Adams’s husband let a store clerk pick out each vase and that each vase was marked at around twelve or fourteen dollars.

The main context clue is right in the phrase in which the underlined word appears: “marked at about twelve to fourteen dollars.” Choice D, “priced,” makes obvious sense here, as what follows the underlined word is a range of costs that Mrs. Adams’s husband paid. (Prices being “marked” on items for sale is also a common way in American English to indicate that these items have price tags of some sort on them, but even without that knowledge, you should easily recognize that “marked” in this context is closely related to price.)

STRATEGY

Incorrect answer choices for Words in Context questions that ask you for a definition, such as this question, will often offer real-world meanings for the target word or phrase (in this case, “marked”), but the key will be the most logical and precise in the provided context.

By contrast, none of the other answer choices makes much sense in context, as it wouldn’t be logical or customary to describe an item for sale as “stained” (choice A), “staged” (choice B), or “watched” (choice C) at about twelve or fourteen dollars. You can mark something by staining it, you can use marks in staging to indicate where actors should stand, and you can “mark” someone or something by observing them or it closely, but none of these options makes sense in relation to a listed price.

Text Structure and Purpose Questions

Text Structure and Purpose questions ask you to analyze the ways authors put texts together and for what reasons. You may be asked to describe the overall structure or purpose of a passage or to determine the impact on a passage of a particular element (e.g., a detail) in it.

Let’s consider an example focusing on a passage-level main purpose.

Craft and Structure Sample Question 4

In 1973, poet Miguel Algarín started inviting other writers who, like him, were Nuyorican—a term for New Yorkers of Puerto Rican heritage—to gather in his apartment to present their work. The gatherings were so well attended that Algarín soon had to rent space in a cafe to accommodate them. Thus, the Nuyorican Poets Cafe was born. Moving to a permanent location in 1981, the Nuyorican Poets Cafe expanded its original scope beyond the written word, hosting art exhibitions and musical performances as well. Half a century since its inception, it continues to foster emerging Nuyorican talent. Which choice best describes the overall purpose of the text?

- A) To explain what motivated Algarín to found the Nuyorican Poets Cafe
- B) To situate the Nuyorican Poets Cafe within the cultural life of New York as a whole
- C) To discuss why the Nuyorican Poets Cafe expanded its scope to include art and music
- D) To provide an overview of the founding and mission of the Nuyorican Poets Cafe

Main purpose questions are essentially central idea questions but with a focus on *why* rather than *what*. Like questions about passages' central ideas, main purpose questions require you to understand the passage as a whole and to provide a summary that incorporates the key ideas without leaving anything critical out or including anything extraneous. The difference is that main purpose questions address authors' intentions, or their aims for writing particular texts. Whereas central ideas questions will have answer choices that are (correct or incorrect) statements about the information and ideas contained in passages, each answer choice in main purpose questions will often begin with a verb (and the particle *to*)—"to explain," "to describe," "to argue," and so on.

This passage narrates the development of the Nuyorican Poets Cafe from its informal founding in 1973 as a workshop for poets to its move in 1981 to a permanent location with an expanded focus on other art forms to its continued support for emerging artists over half a century. Choice D provides the best answer to the question, as the intent of the passage is to provide an overview of the birth and goal of the Nuyorican Poets Cafe.

Choice A is incorrect because Algarín's motivation for founding the cafe isn't specifically discussed in the passage and because the passage continues on from the cafe's founding to talk about its expansion and evolution. Choice B is incorrect because although the passage does mention that the cafe "continues to foster emerging Nuyorican talent," it doesn't do so in enough detail to qualify as situating (contextualizing) the cafe within New York City's cultural life, as the passage's focus is on the cafe itself. Choice C is incorrect because the passage doesn't tell us why the cafe expanded its scope to include art and music, only that it did so. In any case, the passage's overall focus is broader, addressing the cafe itself, its beginnings, and its development over time.

The important thing to note here is that, like central ideas questions, main purpose questions require you to not only determine what's factually correct per the passage but also differentiate the main purpose from subordinate, or minor, purposes. The incorrect answers in this example are primarily wrong on factual grounds (i.e., they aren't supported in the passage), but other main purpose questions—particularly more challenging ones—may ask you to distinguish between main and subordinate purposes. For example, an answer choice such as "To explain why Algarín had to rent space in a cafe for his gatherings of writers" would be true to the facts of the passage—which tells us that Algarín had to rent cafe space because the gatherings were so well attended—but would represent a subordinate purpose of the passage rather than the main purpose, since that explanation is merely one part of the larger account of the Nuyorican Poets Cafe that the passage provides.

REMINDER

Main purpose questions call on the same general skill as central ideas questions. You're looking for the answer choice that best captures the whole passage.

STRATEGY

A key skill required to answer some main purpose questions, especially more challenging ones, is the ability to distinguish the *main* purpose from other, smaller purposes the passage might also be trying to accomplish (i.e., *subordinate* purposes).

While we're thinking about entire passages from a rhetorical standpoint, let's now consider an example about the overall structure of a passage.

Craft and Structure Sample Question 5

Many films from the early 1900s have been lost. These losses include several films by the first wave of Black women filmmakers. We know about these lost movies only from small pieces of evidence. For example, an advertisement for Jennie Louise Touissant Welcome's documentary *Doing Their Bit* still exists. There's a reference in a magazine to Tressie Souder's film *A Woman's Error*. And Maria P. Williams's *The Flames of Wrath* is mentioned in a letter and a newspaper article, and one image from the movie was discovered in the 1990s.

Which choice best describes the overall structure of the text?

- A) The text discusses several notable individuals, then explains commonly overlooked differences between those individuals.
- B) The text describes a general situation, then illustrates that situation with specific examples.
- C) The text identifies a complex problem, then presents examples of unsuccessful attempts to solve that problem.
- D) The text summarizes a debate among researchers, then gives reasons for supporting one side in that debate.

REMINDER

We spent a good deal of time discussing how to analyze the structure of passages in chapter 8.

This question asks us to pick the answer choice that best describes the overall structure of the passage. In chapter 8, we used text structure as one of a few key ways to analyze and make sense of passages. Here, our goal is to describe text structure itself.

The answer choices for this question all follow the same pattern. Each choice presents two ideas linked by the word *then*, which indicates sequence. First the passage does one thing, and then it does another thing.

Before turning to the answer choices, let's break down the passage's structure ourselves. The passage's first three sentences ("Many films . . . evidence") tell us that many early films, including films from "the first wave of Black women filmmakers," have been lost and that we only know those films existed because of "small pieces of evidence." The turning point in the passage is the fourth sentence, which begins with "for example" and introduces a series of cases of early films by Black women of which we have only scant evidence. Our preview of the answer choices led us to anticipate this circumstance, as we saw that each choice consisted of two ideas—two things the passage might do—linked by *then*.

Choice B closely tracks with our analysis of the passage's overall structure. The passage's first three sentences describe a general situation—the loss of many early films and, in particular, the near-total loss of those of many early Black women filmmakers—while the passage's last three sentences provide three examples of early films by Black women of which little trace remains.

Choice A is incorrect because the passage begins with a description of a general situation, not of several notable individuals, and because the passage concludes with examples of the general situation, not a discussion of differences among individuals. Moreover, the individuals mentioned in the passage's last three sentences are more alike than different, as each of their films has all but disappeared. Choice C is incorrect because the first three sentences of the passage present a regrettable set of facts more than a "problem" (complex or otherwise) to be solved and because the last three sentences offer examples illustrating the state of affairs rather than a series of failed solutions. Choice D is incorrect because no debate is presented in the passage.

Now let's examine the third type of Text Structure and Purpose question you may encounter in the Reading and Writing section: questions that ask you to determine the function (role) of part of a passage in relation to the passage as a whole. We'll number the sentences (and the underlined portion) for ease of reference in our discussion, but be aware that sentences in passages aren't numbered in actual test questions.

Craft and Structure Sample Question 6

(1) Horizontal gene transfer occurs when an organism of one species acquires genetic material from an organism of another species through nonreproductive means. (2) The genetic material can then be transferred "vertically" in the second species—that is, through reproductive inheritance. (3) Scientist Atma Ivancevic and her team have hypothesized infection by invertebrate parasites as a mechanism of horizontal gene transfer between vertebrate species: (4) while feeding, a parasite could acquire a gene from one host, then relocate to a host from a different vertebrate species and transfer the gene to it in turn.

Which choice best describes the function of the underlined portion in the text as a whole?

- A) It explains why parasites are less susceptible to horizontal gene transfer than their hosts are.
- B) It clarifies why some genes are more likely to be transferred horizontally than others are.
- C) It contrasts how horizontal gene transfer occurs among vertebrates with how it occurs among invertebrates.
- D) It describes a means by which horizontal gene transfer might occur among vertebrates.

Let's first break down the structure of this passage as a whole and then zero in on the role the underlined portion plays in the passage.

As with many opening sentences in Reading and Writing passages (and in real-world texts), sentence 1 here provides background—in this case, a brief explanation of the process of horizontal gene transfer. Sentence 2 continues the explanation by contrasting horizontal gene transfer with vertical transfer. From these sentences, we learn that while vertical gene transfer occurs through reproduction, horizontal gene transfer doesn't. Sentence 3 (before the colon) states the

STRATEGY

To answer questions about the function of parts of passages, you'll want to have a good sense of what the passage as a whole is trying to accomplish. The answer will be the main contribution the tested part of the passage makes to the whole passage.

hypothesis of Ivancevic and colleagues, which involves invertebrate parasites serving as a mechanism (means) for horizontal gene transfer between vertebrate species. In some sense, the parasites, the researchers speculate, act as a genetic “bridge” between other species.

The underlined portion, which we’ve numbered 4, is the part of the passage for which we need to determine a function in relation to the passage as a whole. In other words, we need to identify what the underlined portion mainly contributes to the passage.

Through careful reading, we can recognize that the underlined portion builds on sentence 3 by further explaining the mechanism posited by the researchers. There are several clues leading us to this interpretation. First, and most obviously, the content of the underlined portion describes a possible method by which parasites could be responsible for horizontal gene transfer: by gaining a gene from one host, moving to another host, and passing on the gene to the second host. Our second clue is the use of the colon in sentence 3. Colons can be used to introduce lists, for example, but their other main use in writing (especially in academic writing, which the Reading and Writing section’s passages exemplify) is to introduce an elaboration. *Elaborations* are word(s), phrase(s), or even sentence(s) that build on or clarify the meaning of prior statements—in this case, the statement before the colon marked as sentence 3. This is exactly the role of the underlined portion in this passage. What follows the colon is an extension and clarification of what’s said in sentence 3. While sentence 3 describes the way horizontal gene transfer involving parasites may occur, the underlined portion adds specific details about how that mechanism may work. A final clue to the role of sentence 4 is the use of the conditional mood verb “could acquire” in the underlined portion. “Could” here signals that what’s being described in the underlined portion is a possible way in which horizontal gene transfer involving parasites may occur. This ties back nicely to sentence 3, which tells us that the researchers had a hypothesis about how parasites may facilitate horizontal gene transfer.

Choice D turns out to match very closely with the reasoning we just detailed and is the best answer to this question. Choice A is incorrect because it’s not factually accurate. The passage does describe how parasites could be involved in horizontal gene transfer, but nothing in the passage suggests that parasites are “less susceptible” to such transfers, only that the parasites might facilitate such transfers. Choice B is incorrect because the passage doesn’t tell us anything about which genes are capable of or prone to being transferred horizontally. Choice C is incorrect because the passage talks only about horizontal gene transfer among vertebrate species; the only reference to invertebrates is to the parasites themselves, and the only real comparison the passage makes is between horizontal and vertical gene transfer.

Cross-Text Connections Questions

Let's close out this chapter with the third and final main type of Craft and Structure question: questions that require you to make supportable text-based conclusions about how two passages on the same topic or similar topics relate to one another.

In **Cross-Text Connections** questions, you'll be presented with two brief passages that are related by topic. After reading both passages, you'll have to determine which of the question's answer choices is best supported by evidence in each of the passages. Individual questions of this type may focus on similarities or differences between the two passages, and these relationships may be fairly straightforward or quite subtle.

Let's consider some examples.

Craft and Structure Sample Question 7

Text 1

Today the starchy root cassava is found in many dishes across West Africa, but its rise to popularity was slow. Portuguese traders brought cassava from Brazil to the West African coast in the 1500s. But at this time, people living in the capitals further inland had little contact with coastal communities. Thus, cassava remained relatively unknown to most of the region's inhabitants until the 1800s.

Text 2

Cassava's slow adoption into the diet of West Africans is mainly due to the nature of the crop itself. If not cooked properly, cassava can be toxic. Knowledge of how to properly prepare cassava needed to spread before the food could grow in popularity. The arrival of formerly enslaved people from Brazil in the 1800s, who brought their knowledge of cassava and its preparation with them, thus directly fueled the spread of this crop.

Based on the texts, the author of Text 1 and the author of Text 2 would most likely agree with which statement?

- A) The climate of the West African coast in the 1500s prevented cassava's spread in the region.
- B) Several of the most commonly grown crops in West Africa are originally from Brazil.
- C) The most commonly used methods to cook cassava today date to the 1500s.
- D) Cassava did not become a significant crop in West Africa until long after it was first introduced.

Our general approach to this question will be similar to how we've tackled questions about single passages. We'll first consider what each passage says separately and then look for one or more supportable connections between the two texts.

The central idea of Text 1 is stated directly in the passage's last sentence: cassava remained relatively unknown in West Africa until the nineteenth century. In terms of purpose, Text 2 mainly serves to explain why cassava remained underutilized in West Africa until

STRATEGY

In many ways, Cross-Text Connections questions can be approached like other questions asking only about a single passage. You'll want to begin by getting a clear sense of the message of each passage separately as well as of the point of view or perspective expressed in each passage.

STRATEGY

Once you've grasped the message and point of view of each passage, think about how the two passages intersect. The second passage may, for example, offer details not included in the first passage, offer an example of what's described in the first passage, or support or challenge all or part of the first passage. In each case, the question itself will help guide you, as it will signal the kind of relationship you should be seeking.

the 1800s and why the situation changed with the arrival of formerly enslaved people from Brazil who knew how to prepare cassava safely. The two passages together give us a clearer view of the emergence of cassava in West Africa.

Now let's consider the question we're posed. Among the answer choices, we want to find the one that the author of Text 1 and the author of Text 2 would most likely agree on. Before we examine the answer choices closely, let's think about how the information in and the perspectives of the two passages relate. As we already noted, Text 1 focuses on the fact that cassava didn't become well known in West Africa until the 1800s, while Text 2 focuses on why cassava only emerged as an important West African food source in the 1800s. The two texts, in other words, are complementary, as they each present different parts of the story of cassava's introduction into the West African diet. Although the two passages contain different details, they agree on the main points.

Choice D is the best answer because it states a point that both authors would likely agree with. Both passages indicate that a lot of time passed between the introduction of cassava to West Africa and its widespread use as a source of food. Text 1 tells us directly that cassava was introduced to West Africa in the 1500s. Text 2 doesn't dispute that estimate and also characterizes the adoption of cassava as a food source in West Africa as "slow." In addition, Text 1 tells us directly that cassava "remained relatively unknown to most of the region's inhabitants until the 1800s," while Text 2 indicates that cassava didn't take off as a food source in West Africa until the arrival of formerly enslaved people from Brazil in the 1800s. Thus, both passages agree that it took a long time for cassava to become important to West Africans' diets.

Choice A, on the other hand, can be ruled out because neither passage suggests that climate prevented the spread of cassava: Text 1 suggests that the distance from the coasts to the inland capitals impeded the spread of cassava, while Text 2 suggests that cassava's toxicity kept it from being widely adopted as a food source. The passages also tell us only about a single crop, not several crops, coming to West Africa from Brazil, which rules out choice B. Text 2 establishes that successful methods for cooking cassava in West Africa emerged only in the 1800s with the arrival of formerly enslaved people from Brazil, which rules out choice C.

In the preceding example, only one answer choice, the key, was supported by textual evidence from the passages. In more challenging questions of this type, such as the following example, you'll have to consider more plausible choices that require more reasoning to dismiss.

Craft and Structure Sample Question 8

Text 1

Although food writing is one of the most widely read genres in the United States, literary scholars have long neglected it. And within this genre, cookbooks attract the least scholarly attention of all, regardless of how well written they may be. This is especially true of works dedicated to regional US cuisines, whose complexity and historical significance are often overlooked.

Text 2

With her 1976 cookbook *The Taste of Country Cooking*, Edna Lewis popularized the refined Southern cooking she had grown up with in Freetown, an all-Black community in Virginia. She also set a new standard for cookbook writing: the recipes and memoir passages interspersing them are written in prose more elegant than that of most novels. Yet despite its inarguable value as a piece of writing, Lewis's masterpiece has received almost no attention from literary scholars.

Based on the two texts, how would the author of Text 1 most likely regard the situation presented in the underlined sentence in Text 2?

- A) As typical, because scholars are dismissive of literary works that achieve popularity with the general public
- B) As unsurprising, because scholars tend to overlook the literary value of food writing in general and of regional cookbooks in particular
- C) As justifiable, because Lewis incorporated memoir into *The Taste of Country Cooking*, thus undermining its status as a cookbook
- D) As inevitable, because *The Taste of Country Cooking* was marketed to readers of food writing and not to readers of other genres

Recall that in our first Cross-Text Connections example, the two passages were quite similar in overall focus and perspective, differing only in details. In this example, the relationship between the two passages is more subtle and thus requires more reasoning to infer.

After reading both passages, we can say in broad terms that Text 1 describes a general state of affairs and that Text 2 provides a concrete example of that state. But there's a little bit more going on than that, and the text-based connection we're being asked to draw between the two passages requires a finer-level understanding of both passages.

Let's consider Text 1 first. The author describes a nested set of relationships concerning the esteem (or, rather, lack thereof) in which scholars hold certain forms of writing. Food writing, the broadest category of the three mentioned in Text 1, is "neglected." Within the category of food writing, cookbook writing is particularly devalued. And within the category of cookbook writing, regional cookbook writing is the most devalued. If we wanted to write this out using basic math symbology, we could come up with the following, which uses "greater than" signs to indicate the hierarchy described in Text 1.

(Other, esteemed forms of writing) > food writing > cookbook writing > regional cookbook writing

NOTE

In more challenging Cross-Text Connections questions, both the passages and the relationship between them tend to become more subtle and complex.

STRATEGY

While we strongly recommend that you read and consider the entirety of each answer choice to a given question, it's possible that some choices can be ruled out quickly based on (as here) a single key word or phrase that's clearly incorrect.

Once we've mapped out the relationships described in Text 1 (whether literally or in our heads), it's a fairly straightforward task to understand the intended connection to Text 2. Text 2 tells us that Edna Lewis published *The Art of Country Cooking*, a regional cookbook of "inarguable value," in 1976. Based only on our analysis of Text 1, we would anticipate that Lewis's cookbook wasn't highly valued by scholars, as regional cookbook writing was lowest in the hierarchy described in Text 1. This inference is borne out by evidence in Text 2, which tells us, in the underlined (last) sentence, that "Lewis's masterpiece has received almost no attention from literary scholars."

Let's consider now the answer choices. Each choice consists of two parts, which we might call a *characterization* and a *reason*. From the question itself, we learn that the characterizations—"typical," "unsurprising," "justifiable," and "inevitable"—are posed as possible reactions of the author of Text 1 to the situation described in the underlined sentence of Text 2 (Lewis's cookbook has been ignored by scholars despite its obvious quality). Even before we get to the reasons that follow each characterization, we can easily rule out choice C based on "justifiable." Nothing suggests that the author of Text 1 would regard scholarly neglect of a landmark regional cookbook as justified, as the author is critical of the scholarly neglect of cookbooks in general and regional cookbooks in particular.

To choose the best answer among the remaining choices, we need to consider the reasons—the "because" clauses—provided in each. In doing so, choice B turns out to be the best answer to the question. The underlined portion in Text 2 asserts that Lewis's work has been unfairly overlooked. Based on our analysis of the passages, it's reasonable to conclude that the author of Text 1 would find this neglect unsurprising, and the reason given in choice B—"because scholars tend to overlook the literary value of food writing in general and of regional cookbooks in particular"—tracks with our breakdown of the hierarchy of relationships depicted in Text 1.

STRATEGY

When answering Reading and Writing questions, especially more challenging ones, it's critical to distinguish between what *may* be possible or even plausible in the real world and what's actually supported in the passage(s). The key will be the answer choice with the firmest grounding in the passage(s).

Choice A is incorrect because while the characterization "typical" carries about the same meaning as "unsurprising" in choice B, the reason given in choice A is unsupported by Text 1. The author of Text 1 does mention that food writing is "one of the most widely read genres in the United States" but doesn't offer evidence that food writing is dismissed by scholars just because it's popular. It's *possible* that the author of Text 1 believes critics are biased against popular forms of writing, but the text doesn't directly say or strongly imply that, so we can safely rule out this answer option. As we've observed, the characterization "justifiable" in choice C immediately rules out this option, but we can also note that the reason provided—Lewis undermined the status of *The Taste of Country Cooking* by incorporating memoir into it—has no support in either text. Choice D is incorrect because the characterization "inevitable" is a little stronger

than Text 1 supports and because there's no evidence in either text to support the assertion that *The Taste of Country Cooking* was marketed to readers of food writing and not to readers of other genres.

In our final example for this chapter, we'll examine a Cross-Text Connections question that, unlike the other two discussed, asks you to identify a difference in perspective between two passages.

Craft and Structure Sample Question 9

Text 1

Despite its beautiful prose, *The Guns of August*, Barbara Tuchman's 1962 analysis of the start of World War I, has certain weaknesses as a work of history. It fails to address events in Eastern Europe just before the outbreak of hostilities, thereby giving the impression that Germany was the war's principal instigator. Had Tuchman consulted secondary works available to her by scholars such as Luigi Albertini, she would not have neglected the influence of events in Eastern Europe on Germany's actions.

Text 2

Barbara Tuchman's *The Guns of August* is an engrossing if dated introduction to World War I. Tuchman's analysis of primary documents is laudable, but her main thesis that European powers committed themselves to a catastrophic outcome by refusing to deviate from military plans developed prior to the conflict is implausibly reductive.

Which choice best describes a difference in how the authors of Text 1 and Text 2 view Barbara Tuchman's *The Guns of August*?

- A) The author of Text 1 believes that the scope of Tuchman's research led her to an incorrect interpretation, while the author of Text 2 believes that Tuchman's central argument is overly simplistic.
- B) The author of Text 1 argues that Tuchman should have relied more on the work of other historians, while the author of Text 2 implies that Tuchman's most interesting claims result from her original research.
- C) The author of Text 1 asserts that the writing style of *The Guns of August* makes it worthwhile to read despite any perceived deficiency in Tuchman's research, while the author of Text 2 focuses exclusively on the weakness of Tuchman's interpretation of events.
- D) The author of Text 1 claims that Tuchman would agree that World War I was largely due to events in Eastern Europe, while the author of Text 2 maintains that Tuchman would say that Eastern European leaders were not committed to military plans in the same way that other leaders were.

Before we consider differences between the perspectives of the authors of the two passages in this question, let's identify some of the ways in which the two texts are similar in their views. Both authors describe *The Guns of August* as well written, with the author of Text 1 saying it has "beautiful prose" and the author of Text 2 calling it "engrossing." In addition, both authors agree that *The Guns of August* has serious limitations as a work of history. Text 1 argues that Tuchman paid too little attention to prewar events in Eastern Europe, while Text 2 contends that Tuchman gave too much credence to the idea that a refusal to deviate from military plans made before the outbreak of hostilities caused World War I.

STRATEGY

Authors of passages in Cross-Text Connections questions may agree or disagree in whole or in part. More challenging questions of this type will pair passages with subtle or complex relationships that you must use reading, reasoning, and text-based inferring to determine.

This discussion leads us to an important general point about Cross-Text Connections passages. Even when the authors of such passages disagree, this disagreement may only be partial. You may see pairs of passages in the Reading and Writing section in which two clearly opposing viewpoints are contrasted, but that's not always going to be the case, especially with more challenging questions of this type.

Choice A is the best answer to this question, as it captures an important difference between the two passages. The author of Text 1 objects to Tuchman's lack of attention to prewar events in Eastern Europe and attributes this to her failure to consult appropriate secondary sources. We can fairly conceptualize this, as choice A does, as a narrowness in the scope of Tuchman's work (i.e., not adequately considering secondary sources) led Tuchman to an incorrect interpretation (i.e., Germany was the war's primary instigator). We find support for the other half of choice A in Text 2, in which the author refers to Tuchman's central argument as "implausibly reductive." Even if the precise meaning of *reductive* (overly simplistic) eludes you, the adverb *implausibly* should clue you in to choice A being the best option.

Choice B is incorrect because although the first half of the answer choice is basically accurate—the author of Text 1 does argue that Tuchman should have consulted more secondary sources—the second half of the answer choice isn't really supported by Text 2. The author of Text 2 clearly respects *The Guns of August* at some level, but Text 2's emphasis is on the book's shortcomings as a work of history. Both halves of choice C are incorrect. The author of Text 1, as we've observed, does praise the prose of *The Guns of August*, but the author never says or implies anything as strongly worded as the proposition that the writing quality of the book transcends its weaknesses as history. Furthermore, although the author of Text 2 does focus primarily on the limitations of *The Guns of August* as a historical account, the author also praises the book, calling it "engrossing" and lauding Tuchman's analysis of primary sources. Choice D is incorrect because, like choice C, it's false on both counts. The author of Text 1 claims that Tuchman ignored prewar events in Eastern Europe, and the author of Text 2 never mentions Eastern Europe specifically, only Europe in general.

As we saw with this question, the most challenging Cross-Text Connections questions will require you to carefully evaluate two (potentially complex) passages while also sorting through answer choices that may be partially but not wholly right. But, as we've also shown, you can use the same basic approach to these questions regardless of the passages' content or the specific question you're asked: read each passage closely, and then establish the nature of the relationship between the two passages. Do they largely agree? Do they largely disagree? Do they simply cover different aspects of the same topic? Getting a strong sense of both passages' content and how the two texts relate will give you a solid basis for answering any question about the pairing.

Summary: Craft and Structure Questions

Main purpose: To test whether you can determine how and why authors write texts the way they do

Proportion of the test section: About 28 percent (approximately 13–15 questions)

Question types

- **Words in Context:** Use and determine the meaning of high-utility academic words and phrases in the contexts in which they appear
- **Text Structure and Purpose:** Analyze and describe how and why passages are put together the way they are; describe the main purpose of passages
- **Cross-Text Connections:** Draw supportable connections between two topically related passages



CHAPTER 14

Reading and Writing: Questions—Craft and Structure Drills

1

Biologist Jane Edgeloe and colleagues have located what is believed to be the largest individual plant in the world in the Shark Bay area of Australia. The plant is a type of seagrass called *Posidonia australis*, and it _____ approximately 200 square kilometers.

Which choice completes the text with the most logical and precise word or phrase?

- A) acknowledges
- B) produces
- C) spans
- D) advances

2

Osage Nation citizen Randy Tinker-Smith produced and directed the ballet *Wahzhazhe*, which vividly chronicles Osage history and culture. Telling Osage stories through ballet is _____ choice because two of the foremost ballet dancers of the twentieth century were Osage: sisters Maria and Marjorie Tallchief.

Which choice completes the text with the most logical and precise word or phrase?

- A) an unpredictable
- B) an arbitrary
- C) a determined
- D) a suitable

3

The following text is adapted from Zora Neale Hurston's 1921 short story "John Redding Goes to Sea." John wants to travel far beyond the village where he lives near his mother, Matty.

[John] had on several occasions attempted to reconcile his mother to the notion, but found it a difficult task. Matty always took refuge in self-pity and tears. Her son's desires were incomprehensible to her, that was all.

As used in the text, what does the phrase "reconcile his mother to" most nearly mean?

- A) Get his mother to accept
- B) Get his mother to apologize for
- C) Get his mother to match
- D) Get his mother to reunite with

4

Micheline Pesantubbee, a historian and citizen of the Choctaw Nation, has identified a dilemma inherent to research on the status of women in her tribe during the 1600s and 1700s: the primary sources from that era, travel narratives and other accounts by male European colonizers, underestimate the degree of power conferred on Choctaw women by their traditional roles in political, civic, and ceremonial life. Pesantubbee argues that the Choctaw oral tradition and findings from archaeological sites in the tribe's homeland supplement the written record by providing crucial insights into those roles.

Which choice best describes the overall structure of the text?

- A) It details the shortcomings of certain historical sources, then argues that research should avoid those sources altogether.
- B) It describes a problem that arises in research on a particular topic, then sketches a historian's approach to addressing that problem.
- C) It lists the advantages of a particular research method, then acknowledges a historian's criticism of that method.
- D) It characterizes a particular topic as especially challenging to research, then suggests a related topic for historians to pursue instead.

5

The following text is from Srimati Svarna Kumari Devi's 1894 novel *The Fatal Garland* (translated by A. Christina Albers in 1910). Shakti is walking near a riverbank that she visited frequently during her childhood.

She crossed the woods she knew so well. The trees seemed to extend their branches like welcoming arms. They greeted her as an old friend. Soon she reached the river-side.

Which choice best describes the function of the underlined portion in the text as a whole?

- A) It suggests that Shakti feels uncomfortable near the river.
- B) It indicates that Shakti has lost her sense of direction in the woods.
- C) It emphasizes Shakti's sense of belonging in the landscape.
- D) It conveys Shakti's appreciation for her long-term friendships.

6

The following text is adapted from Oscar Wilde's 1897 nonfiction work *De Profundis*.

People whose desire is solely for self-realisation never know where they are going. They can't know. In one sense of the word it is of course necessary to know oneself: that is the first achievement of knowledge. But to recognise that the soul of a man is unknowable, is the ultimate achievement of wisdom. The final mystery is oneself. When one has weighed the sun in the balance, and measured the steps of the moon, and mapped out the seven heavens star by star, there still remains oneself. Who can calculate the orbit of his own soul?

Which choice best describes the function of the underlined question in the text as a whole?

- A) It cautions readers that the text's directions for how to achieve self-knowledge are hard to follow.
- B) It concedes that the definition of self-knowledge advanced in the text is unpopular.
- C) It reinforces the text's skepticism about the possibility of truly achieving self-knowledge.
- D) It speculates that some readers will share the doubts expressed in the text about the value of self-knowledge.

7

In many agricultural environments, the banks of streams are kept forested to protect water quality, but it's been unclear what effects these forests may have on stream biodiversity. To investigate the issue, biologist Xingli Giam and colleagues studied an Indonesian oil palm plantation, comparing the species richness of forested streams with that of nonforested streams. Giam and colleagues found that species richness was significantly higher in forested streams, a finding the researchers attribute to the role leaf litter plays in sheltering fish from predators and providing food resources.

Which choice best states the main purpose of the text?

- A) It presents a study that addresses an unresolved question about the presence of forests along streams in agricultural environments.
- B) It explains the differences between stream-protection strategies used in oil palm plantations and stream-protection strategies used in other kinds of agricultural environments.
- C) It discusses research intended to settle a debate about how agricultural yields can be increased without negative effects on water quality.
- D) It describes findings that challenge a previously held view about how fish that inhabit streams in agricultural environments attempt to avoid predators.

8

Text 1

Dance choreographer Alvin Ailey's deep admiration for jazz music can most clearly be felt in the rhythms and beats his works were set to. Ailey collaborated with some of the greatest jazz legends, like Charles Mingus, Charlie Parker, and perhaps his favorite, Duke Ellington. With his choice of music, Ailey helped bring jazz to life for his audiences.

Text 2

Jazz is present throughout Ailey's work, but it's most visible in Ailey's approach to choreography. Ailey often incorporated improvisation, a signature characteristic of jazz music, in his work. When managing his dance company, Ailey rarely forced his dancers to an exact set of specific moves. Instead, he encouraged his dancers to let their own skills and experiences shape their performances, as jazz musicians do.

Based on the texts, both authors would most likely agree with which statement?

- A) Audiences were mostly unfamiliar with the jazz music in Ailey's works.
- B) Ailey's work was strongly influenced by jazz.
- C) Dancers who worked with Ailey greatly appreciated his supportive approach as a choreographer.
- D) Ailey blended multiple genres of music together when choreographing dance pieces.

9

Text 1

The idea that time moves in only one direction is instinctively understood, yet it puzzles physicists. According to the second law of thermodynamics, at a macroscopic level some processes of heat transfer are irreversible due to the production of entropy—after a transfer we cannot rewind time and place molecules back exactly where they were before, just as we cannot unbreak dropped eggs. But laws of physics at a microscopic or quantum level hold that those processes *should* be reversible.

Text 2

In 2015, physicists Tiago Batalhão et al. performed an experiment in which they confirmed the irreversibility of thermodynamic processes at a quantum level, producing entropy by applying a rapidly oscillating magnetic field to a system of carbon-13 atoms in liquid chloroform. But the experiment “does not pinpoint . . . what causes [irreversibility] at the microscopic level,” coauthor Mauro Paternostro said.

Based on the texts, what would the author of Text 1 most likely say about the experiment described in Text 2?

- A) It is consistent with the current understanding of physics at a microscopic level but not at a macroscopic level.
- B) It provides empirical evidence that the current understanding of an aspect of physics at a microscopic level must be incomplete.
- C) It supports a claim about an isolated system of atoms in a laboratory, but that claim should not be extrapolated to a general claim about the universe.
- D) It would suggest an interesting direction for future research were it not the case that two of the physicists who conducted the experiment disagree on the significance of its findings.

10

Text 1

Because literacy in Nahuatl script, the writing system of the Aztec Empire, was lost after Spain invaded central Mexico in the 1500s, it is unclear exactly how meaning was encoded in the script's symbols. Although many scholars had assumed that the symbols signified entire words, linguist Alfonso Lacadena theorized in 2008 that they signified units of language smaller than words: individual syllables.

Text 2

The growing consensus among scholars of Nahuatl script is that many of its symbols could signify either words or syllables, depending on syntax and content at any given site within a text. For example, the symbol signifying the word *huipil* (blouse) in some contexts could signify the syllable “pil” in others, as in the place name “Chipiltepec.” Thus, for the Aztecs, reading required a determination of how such symbols functioned each time they appeared in a text.

Based on the texts, how would the author of Text 2 most likely characterize Lacadena's theory, as described in Text 1?

- A) By praising the theory for recognizing that the script's symbols could represent entire words
- B) By arguing that the theory is overly influenced by the work of earlier scholars
- C) By approving of the theory's emphasis on how the script changed over time
- D) By cautioning that the theory overlooks certain important aspects of how the script functioned

Answer Explanations

QUESTION 1

Choice C is the best answer because it completes the text with the most logical and precise word. "Spans" means "extends over a distance of" or "encompasses," which makes sense in describing the area covered by what's believed to be the world's largest individual plant.

Choice A is incorrect because "acknowledges" means "recognizes" or "admits the truth of." Either way, it doesn't make sense here, as a plant can't acknowledge an area. **Choice B** is incorrect because "produces" can mean "makes," "causes," or "presents," but none of those definitions makes sense here, as a plant can't make, cause, or present an area. **Choice D** is incorrect because "advances" means "moves forward" or "progresses," but the plant isn't necessarily moving forward or progressing. Rather, the passage indicates that it already covers an area of 200 square kilometers.

QUESTION 2

Choice D is the best answer because it completes the text with the most logical and precise phrase. "Suitable" means "appropriate for a particular purpose." Since the passage indicates that two of the best ballet dancers of the twentieth century were Osage, we can reasonably infer that the author believes that ballet is a suitable art form for telling Osage stories.

Choice A is incorrect because the passage never suggests that Tinker-Smith's choice was an "unpredictable" one. Rather, the fact that two of the best ballet dancers of the twentieth century were Osage makes ballet especially appropriate for telling Osage stories. **Choice B** is incorrect because the passage implies the opposite. An "arbitrary" choice is a choice based on whim rather than reason, but the passage gives a good reason behind the choice to tell Osage stories through ballet: two of the best ballet dancers of the twentieth century were Osage. **Choice C** is incorrect because the passage never suggests that Tinker-Smith's choice was a "determined" one, which would imply that Tinker-Smith initially faced some kind of obstacle or opposition, or that the choice was made for Tinker-Smith. Nothing like either of those ideas, however, is mentioned in or suggested by the passage.

QUESTION 3

Choice A is the best answer because it most nearly identifies what the underlined phrase means in context. The expression "reconcile . . . to" refers to causing someone to come to accept something difficult or disagreeable. The passage indicates that John wants his mother to accept his desire to travel even though she doesn't like the idea.

Choice B is incorrect because, per the passage, John doesn't want his mother to apologize for his own desire to travel; he just wants her to accept the idea. **Choice C** is incorrect because the passage doesn't suggest that John wants his mother to match, or share, his desire to travel. Rather, he wants her to accept his desire to travel even though she doesn't like the idea. **Choice D** is incorrect because a person can't really "reunite with" a notion, or idea; in any case, "reunite" illogically suggests that John's mother would be once again accepting the notion of John traveling, when the passage gives no indication that she'd previously accepted the idea.

QUESTION 4

Choice B is the best answer because it best describes the overall structure of the text. The passage begins by stating a problem with existing research on the status of Choctaw women during the 1600s and 1700s: because of the nature of their authorship, written primary sources underestimate the power Choctaw women had in their traditional roles. Then it presents one historian's solution: looking to oral tradition and archeological findings for additional insights into these roles.

Choice A is incorrect because the passage never says that research should avoid written primary sources altogether, just that research should also make use of oral tradition and archeological findings as sources. **Choice C** is incorrect because the passage never mentions the advantages of using written primary sources and because it goes beyond critiquing the use of such sources to suggest ways to supplement them. **Choice D** is incorrect because the passage never says that the status of Choctaw women during the 1600s and 1700s is unusually challenging to research compared to other historical topics and because it doesn't mention any other topics for historians to research instead.

QUESTION 5

Choice C is the best answer because it best describes the function of the underlined portion in the text as a whole. The phrase "welcoming arms" reinforces the passage's suggestion that Shakti feels a sense of belonging in the woods "she knew so well," as if the trees are embracing her.

Choice A is incorrect because the phrase "welcoming arms" suggests that Shakti is comfortable, not uncomfortable, in the woods. **Choice B** is incorrect because the phrase "welcoming arms" reinforces the passage's suggestion that the woods are familiar to Shakti, not that she's become lost. **Choice D** is incorrect because the underlined portion doesn't describe actual long-term friendships; it instead uses figurative language to emphasize Shakti's familiarity with the woods.

QUESTION 6

Choice C is the best answer because it best describes the function of the underlined question in the text as a whole. The passage repeatedly claims that true self-knowledge can't be achieved, and the rhetorical question in the underlined portion, which also expresses skepticism, emphasizes that point.

Choice A is incorrect because the passage doesn't provide directions for how to achieve self-knowledge; rather, it claims that true self-knowledge is impossible to achieve. **Choice B** is incorrect because the text doesn't really ever define self-knowledge, much less discuss the popularity of that nonexistent definition. **Choice D** is incorrect because the passage never expresses doubts about the value of self-knowledge; rather, the doubts expressed in the passage are about the possibility of achieving self-knowledge at all. Additionally, there's nothing in the passage to suggest that the author expects some readers to share his views.

QUESTION 7

Choice A is the best answer because it best states the main purpose of the text. The passage first describes an unresolved question: what effect do forests on stream banks have on stream biodiversity? Then the passage presents a study that answers the question: such forests increase stream biodiversity.

Choice B is incorrect because the passage never mentions any specific agricultural environments other than the one Giam and colleagues studied, which was an Indonesian oil palm plantation. *Choice C* is incorrect because the passage never mentions agricultural yields. *Choice D* is incorrect because the passage never mentions any previously held view about how fish in streams in agricultural environments try to avoid predators.

QUESTION 8

Choice B is the best answer because it identifies the statement with which the authors of the two texts would most likely agree. Text 1 states that Ailey had a “deep admiration for jazz music” and that he “helped bring jazz to life for his audiences” with the inclusion of jazz rhythms and beats in his works. Text 2 states that “jazz is present throughout Ailey’s work.”

Choice A is incorrect because neither passage mentions how familiar or unfamiliar audiences were with any aspect of Ailey’s works. *Choice C* is incorrect because neither passage mentions how Ailey’s dancers felt about his approach as a choreographer. *Choice D* is incorrect because neither passage mentions Ailey employing any genre of music other than jazz.

QUESTION 9

Choice B is the best answer because it identifies what the author of Text 1 would most likely say about the experiment described in Text 2. Text 1 describes a puzzle that physicists hadn’t been able to solve: at a microscopic or quantum level, the laws of physics suggest that we should be able to reverse processes that aren’t reversible at a macroscopic level. The researchers discussed in Text 2, however, provided empirical evidence that those processes aren’t reversible even at the quantum level, for reasons the researchers themselves don’t yet understand. It’s reasonable to conclude, then, that the author of Text 1 would say that our understanding of the laws of physics is incomplete since this understanding led to an incorrect assumption.

Choice A is incorrect because the experiment described in Text 2 contradicts, rather than supports, what the physicists mentioned in Text 1 believed to be true at the microscopic or quantum level. Furthermore, per the passage, the experiment described in Text 2 didn’t address the macroscopic level at all.

Choice C is incorrect because neither passage distinguishes between laboratory findings and the way the universe works in general. *Choice D* is incorrect because although Text 2 mentions multiple physicists and that these physicists don’t yet know what causes irreversibility at the microscopic level, there’s no indication in Text 2 that two of the researchers disagree about the significance of their findings.

QUESTION 10

Choice D is the best answer because it identifies how the author of Text 2 would most likely characterize Lacadena’s theory, as described in Text 1. Lacadena’s theory is that Nahuatl script symbols signified syllables, but the consensus described in Text 2 is that they could signify either syllables or full words, depending on the context. The author of Text 2 would, therefore, likely consider Lacadena’s theory too simplistic: it’s missing the importance of context in determining the meaning of the symbols.

Choice A is incorrect because it conflicts with Text 1’s description of Lacadena’s theory, which is that Nahuatl script symbols signified only syllables. *Choice B* is incorrect because it conflicts with Text 1’s description of Lacadena’s theory. Text 1 states that Lacadena’s theory differed from what earlier scholars had believed. *Choice C* is incorrect because neither passage mentions how or even whether the script changed over time.

CHAPTER 15

Reading and Writing: Questions— Expression of Ideas

Reading and Writing questions in the **Expression of Ideas** content domain are designed to test your ability to revise passages in order to improve how information and ideas are conveyed. There are two main areas of focus in Expression of Ideas questions: selectively using and combining provided information and ideas in order to best meet a specified rhetorical (writerly) goal and using the most logical transition word or phrase to connect information and ideas within a passage.

Proportion of the Test Section

About 20 percent of the Reading and Writing section is made up of Expression of Ideas questions. This translates to approximately eight to twelve Expression of Ideas questions per test form.

Overview of Expression of Ideas Question Types

The Expression of Ideas content domain includes two main question types:

- **Rhetorical Synthesis** questions, which test your ability to selectively use and combine provided information and ideas in order to meet specified writerly goals
- **Transitions** questions, which test your ability to provide the most logical transition word or phrase in order to link information and ideas in passages

All questions in this content domain (and in the Reading and Writing section in general) are multiple-choice in format. Each question has a single best answer (*key*) and three incorrect answer choices (*distractors*) that represent common errors students make when answering such questions. All the information needed to answer each question is in the passage, so you should always base your answers on what a given passage says directly and what it strongly implies.

Expression of Ideas Question Types in Detail

In this section, we'll discuss both Expression of Ideas question types in detail. We'll describe the general purpose of each type so that you understand better how to approach such questions on test day. As part of this discussion, we'll walk through some sample questions so that you get a strong sense of the kinds of skills and knowledge required to successfully answer questions of a given type.

WHAT IS "RHETORIC"?

Rhetoric is simply the art of effective writing (and speaking). You may sometimes find people referring to empty arguments and insincere flowery language as "mere rhetoric," but the term has a broader and more positive meaning as well.

NOTE

Passages for Rhetorical Synthesis questions don't include the literature subject area.

Rhetorical Synthesis Questions

Rhetorical Synthesis questions ask you to selectively use and combine provided information and ideas into an effective single sentence that meets the writerly aim specified in the question itself.

A "passage" for a Rhetorical Synthesis question is made up of a bulleted list of statements about a topic. As the question reminds you, these statements are meant to simulate the kinds of notes a student like you might take during a reading assignment or research project. You'll draw on information and ideas in the statements to answer the question, but you'll need to do so selectively because—as is often the case when you take notes—not all the information and ideas may be needed to meet the writerly goal set forth in the question. You'll have to pay close attention to this goal because it will strongly influence how you answer the question. The answer choices in Rhetorical Synthesis questions will consist of single sentences that present differing ways to combine the information and ideas in the passage, but only one of these ways will best meet the specified writerly goal.

Let's begin with a relatively straightforward Rhetorical Synthesis example. Before going through how to answer the question itself, we'll point out the key features of questions of this type.

Expression of Ideas Sample Question 1

While researching a topic, a student has taken the following notes:

- The painter Frida Kahlo is one of the most influential artists of the twentieth century.
- She was born in Coyoacán, Mexico, in 1907.
- She is best known for her vivid and richly symbolic self-portraits.
- *The Two Fridas* (1939) features two versions of Kahlo sitting together.
- One version wears a European-style dress and the other a traditional Tehuana dress.

The student wants to introduce Kahlo to an audience unfamiliar with the artist. Which choice most effectively uses relevant information from the notes to accomplish this goal?

- A) The 1939 painting *The Two Fridas* is one example of a self-portrait by Frida Kahlo.
- B) One painting by Frida Kahlo features two versions of herself, with one version wearing a European-style dress and the other a traditional Tehuana dress.
- C) Known for being vivid and richly symbolic, Frida Kahlo's self-portraits include *The Two Fridas* (1939).
- D) One of the most influential artists of the twentieth century, Mexican painter Frida Kahlo is best known for her self-portraits, which are vivid and richly symbolic.

Rhetorical Synthesis questions have several important features.

- **Framing.** The framing of the question, or how it's presented to you, asks you to approach this kind of question as if you were a student taking notes for an assignment. Although this framing isn't really necessary to answering the question, it helps to show you how answering the question is like something you've done many times already (and will continue to do) in your classes.
- **Bulleted list.** The statements in the bulleted list represent the kinds of notes you might take as you read about a topic. Just like real notes, this list includes information and ideas relevant to the specified writing goal and may include additional information and ideas that aren't particularly relevant to that goal. As the question itself reminds you, your job will be, in part, to distinguish between relevant and irrelevant information and ideas when answering. You can safely assume that all the information and ideas in these notes are accurate, and therefore you won't be asked to distinguish what's true from what's not.
- **Rhetorical goal.** The rhetorical (writerly) goal specified in the question is critically important to pay attention to. There are many potential ways that the information and ideas in the question's bulleted list *could* be combined into a single sentence, but only one of the four answer choices will do so in a way that most effectively meets the specified goal.

STRATEGY

Pay close attention to the writerly goal specified in Rhetorical Synthesis questions. There are many possible ways to combine the information and ideas in the bulleted list, but only one answer choice will do so in a way that best meets this goal.

NOTE

Rhetorical Synthesis questions don't test grammar, usage, and punctuation. All answer choices will be grammatical. Your job is to focus on meeting the specified writerly goal most effectively, not correcting sentence-level errors. (You'll do the latter in Standard English Conventions questions, the topic of subsequent chapters.)

STRATEGY

Before or while reading Rhetorical Synthesis answer choices, think about what meeting the writerly goal specified in the question might look like—in this case, how you might go about introducing an artist to an audience unfamiliar with her paintings.

- *Answer choices.* Each answer choice represents one possible way to use and combine information and ideas from the notes in the bulleted list. Each sentence is grammatical, and you won't be tested on conventions of sentence structure, usage, and punctuation in this type of question. Instead, you'll have to carefully consider the answer choices to determine which of them best meets the specified writerly goal.

Let's now turn to the particulars of this question.

The five statements in the bulleted list represent notes that a (hypothetical) student has taken about Frida Kahlo and some of her paintings. Before we set about trying to blend this information together, though, we need to consider the writerly goal specified in the question. In this case, the student wants to “introduce Kahlo to an audience unfamiliar with the artist.”

This goal will shape how we evaluate the answer choices. We're not simply searching for the sentence among the answer choices that reads best to us but rather trying to meet the student's goal most effectively. Before (or at least while) reading the actual answer choices carefully, think about what it might mean to meet this goal. In this case, if you were to tell your classmates about an artist you'd researched, and you had good reason to think your fellow students wouldn't know who this artist was, you'd probably want to provide some general information about that artist to orient your listeners. You wouldn't just jump in by, say, describing some of this artist's works but instead would want to give your audience some sense of who the artist was, why they're important (to you or to others), and what they're known for.

Choice D, the best answer to this question, does all these things. It orients and informs readers in general terms about Kahlo and her artistry in a way suitable for an audience unfamiliar with her. This choice tells us that Kahlo was highly influential, worked in the twentieth century, was Mexican, and is best known for her self-portraits, and then characterizes her self-portraits as “vivid and richly symbolic.”

Choice A is incorrect because it's simply a factual statement about one of Kahlo's paintings and doesn't provide any background about Kahlo, as the goal requires. Choice B is incorrect for a similar reason, as it jumps right to discussing one of Kahlo's paintings without any effort at an introduction. Choice C is incorrect as well. It gets closer than either choice A or choice B to offering an introductory statement, but it's a general statement about Kahlo's self-portraits, not, as the goal requires, about Kahlo herself, and the more successful choice D includes the same sort of information about Kahlo's self-portraits as choice C highlights.

Before we turn to another example, let's examine, statement by statement, how the information in the bulleted list was used (or not used) in the key to this question.

Bulleted-list statements	Use in the key
The painter Frida Kahlo is one of the most influential artists of the twentieth century.	This information is fully incorporated into the key.
She was born in Coyoacán, Mexico, in 1907.	The key uses the information that Kahlo was Mexican, but it leaves out her birthplace and the year she was born.
She is best known for her vivid and richly symbolic self-portraits.	This information is fully incorporated into the key.
<i>The Two Fridas</i> (1939) features two versions of Kahlo sitting together.	This information is left out of the key.
One version wears a European-style dress and the other a traditional Tehuana dress.	This information is left out of the key.

As you'll notice, although some information from the statements is used directly in the key, other pieces of information in the bulleted list are left out entirely, while one piece was only partly incorporated. This is normal: part of the task in Rhetorical Synthesis questions is to distinguish relevant from irrelevant information and ideas in terms of meeting the specified writerly goal.

We already implied this, but it's worth singling out and making more explicit: the best answer to a Rhetorical Synthesis question may, and likely will, selectively incorporate and/or paraphrase the information and ideas in its bulleted list. Although the first and the third bullets in the above example appear in the key nearly word for word, this won't always be the case—particularly for more challenging Rhetorical Synthesis questions, which may make greater use of paraphrasing. In addition, as with the second statement in the bulleted list, the best answer may use only part of the information found in some bulleted points. As we noted, Kahlo's Mexican heritage was relevant to meeting the writerly goal specified in the question, but her birthplace and the year she was born were extraneous details that wouldn't have contributed to the most effective introductory statement (choice D). And the best answer may, as we found in this example, leave out some bulleted statements entirely. In this case, the details about Kahlo's painting *The Two Fridas* were omitted because they were too specific and precise for an introductory statement about Kahlo herself.

Our second Rhetorical Synthesis example also highlights the need when answering this type of question to distinguish relevant from irrelevant information and ideas in order to meet the specified writerly goal.

STRATEGY

The best answer to a Rhetorical Synthesis question does *not* have to—and typically won't—use all the information and ideas found in its bulleted list. It may leave out some statements entirely, paraphrase elements of some statements (putting them into different words while still preserving the original meaning), and/or incorporate only the most relevant portions of some statements while leaving out less relevant portions.

Expression of Ideas Sample Question 2

While researching a topic, a student has taken the following notes:

- A wok is a cooking pan that originated in China during the Han dynasty (206 BCE–220 CE).
- The wok's round, wide base helps to cook food evenly.
- The wok's high, angled sides help to contain oil splatters.
- Grace Young is a cook and culinary historian.
- Her book *The Breath of a Wok* (2004) traces the history of the wok.

The student wants to describe the wok's shape. Which choice most effectively uses relevant information from the notes to accomplish this goal?

- A) Grace Young's 2004 book, *The Breath of a Wok*, traces the history of the cooking pan.
- B) A wok is a cooking pan with a round, wide base and high, angled sides.
- C) The design of a wok, a type of cooking pan that originated in China during the Han dynasty, helps the pan cook food evenly and contain oil splatters.
- D) Able to cook food evenly and contain oil splatters, the wok is the subject of Grace Young's 2004 book.

STRATEGY

Don't be afraid to leave out irrelevant information—even one or more whole statements—when answering Rhetorical Synthesis questions. Always keep the specified writerly goal in mind. You may even want to highlight or otherwise annotate the goal before answering to remind you of it.

Two of the five statements in the bulleted list pertain to Grace Young and her history of the wok. Given that forty percent of the information in the list is about Young and her writing, you might reasonably expect that the best answer would include that information somehow. But read the writerly goal closely. The student wants to describe the shape of a wok. Reading the last two sentences in the bulleted list again, we note that knowing that Young is a cook and culinary historian and that she wrote a history of the wok wouldn't add anything to a description of a wok's shape. It's interesting information, certainly, but it's not relevant to answering this particular question.

Choice B is the best answer because it sticks to describing the shape of a wok—it has a “round, wide base” and “high, angled sides.” Choice A is incorrect because it focuses on Young's book and doesn't describe the shape of a wok. Choice C is incorrect because although it focuses on woks themselves, it doesn't describe the shape of a wok at all. Choice D is incorrect because it doesn't describe the shape of a wok and because it focuses as much on Young's book as it does on woks themselves.

Let's look at one final example.

Expression of Ideas Sample Question 3

While researching a topic, a student has taken the following notes:

- As engineered structures, many bird nests are uniquely flexible yet cohesive.
- A research team led by Yashraj Bhosale wanted to better understand the mechanics behind these structural properties.
- Bhosale's team used laboratory models that simulated the arrangement of flexible sticks into nest-like structures.
- The researchers analyzed the points where sticks touched one another.
- When pressure was applied to the model nests, the number of contact points between the sticks increased, making the structures stiffer.

The student wants to present the primary aim of the research study. Which choice most effectively uses relevant information from the notes to accomplish this goal?

- A) The researchers used laboratory models that simulated the arrangement of flexible sticks and analyzed the points where sticks touched one another.
- B) As analyzed by Bhosale's team, bird nests are uniquely flexible yet cohesive engineered structures.
- C) Bhosale's team wanted to better understand the mechanics behind bird nests' uniquely flexible yet cohesive structural properties.
- D) After analyzing the points where sticks touched, the researchers found that the structures became stiffer when pressure was applied.

This example again requires us to carefully differentiate relevant from irrelevant information in terms of the specified writerly goal. Given that this goal is to present the primary aim of the research study, we should be on the lookout for information about that in the bulleted list. The second statement in the list speaks directly to the researchers' motivation: they "wanted to better understand the mechanics behind" the structural properties of bird nests. We should, therefore, expect to find this statement strongly represented in the best answer.

The best answer to this question, choice C, does, in fact, include the second statement from the list, pairing it with the first statement's characterization of birds' nests as "uniquely flexible but cohesive."

Choice A is incorrect because it describes the researchers' approach during the study (i.e., their methodology), not their motivation for undertaking the study in the first place. Choice B is incorrect because it describes the nature of birds' nests, not the researchers' motivation for studying them. Choice D is incorrect because it describes a finding from the research, not the reason the researchers undertook the study.

STRATEGY

We've noted this before, but we think it's worth repeating: it's a good idea when answering Reading and Writing questions to have a sense of what the best answer ought to be (based on a close reading of the associated passage) before carefully examining the answer choices. Doing so may speed up your test taking and will likely make it easier for you to avoid being distracted by incorrect choices.

Transitions Questions

The other main type of Expression of Ideas question focuses on transitions, or the logical links between information and ideas in passages. In real-world texts, transitions serve to guide and smooth readers' movement between and among sentences, paragraphs, and larger sections of texts (such as chapters). In the Reading and Writing section, however, **Transitions** questions are focused on logical connections between and among sentences in brief passages. Let's study a Transitions example, returning to our strategy of analyzing the underlying structure of a passage to examine the relationship between the information and ideas presented and, more specifically, how the sentences themselves relate to one another.

COMMON TRANSITION WORDS AND PHRASES: AN INCOMPLETE LIST

Transitions questions in the Reading and Writing section make use of numerous common transition words and phrases. In linguistic terms, these take the form of conjunctions (e.g., *so*, *but*, *because*) and conjunctive adverbs (e.g., *however*, *as such*, *be that as it may*). Below is a list of some such transitions, though the list is only partial.

Accordingly	For example (for instance)	Moreover
Additionally	For that reason	Nevertheless (nonetheless)
After all	Furthermore	On the contrary
As a result	Hence	On the other hand
As such	In addition	So
Be that as it may	In conclusion (in the end)	Specifically
Because	In other words	That is
But	In summary (in sum)	Therefore
By contrast (in contrast)	(to sum up)	Thus
Consequently	In the meantime	To be sure
Conversely	indeed	Yet
First (second, third, etc.)	Likewise	

Expression of Ideas Sample Question 4

(1) Researchers believe that pieces of hull found off Oregon's coast are from a Spanish cargo ship that was lost in 1697. (2) Stories passed down among the area's Confederated Tribes of Siletz Indians support this belief. (3) _____ Siletz stories describe how blocks of beeswax, an item the ship had been carrying, began washing ashore after the ship was lost.

Which choice completes the text with the most logical transition?

- A) For this reason,
- B) For example,
- C) However,
- D) Likewise,

In our analysis, we'll omit some details to focus on the role each sentence serves. We've numbered the sentence for ease of reference, but they won't be so marked on test day.

- Sentence 1: Researchers make a claim about a discovered ship hull.
- Sentence 2: Researchers find evidence for the claim in stories.
- Sentence 3: ?

Sentence 2's main function is to provide evidence for the researchers' claim about the ship hull found off Oregon's coast. The passage makes this explicit by noting that stories of the Confederated Tribes of Siletz Indians "support" the researchers' claim.

What, then, is the role of sentence 3? Even with the blank, we can tell that that sentence offers additional detail about how the stories support the researchers' claim. It notes that stories about blocks of beeswax appearing on the shore align with the fact that the wrecked ship was carrying beeswax as cargo. In other words, sentence 3 provides a particular case in which the Siletz Indians' stories (sentence 2) support the researchers' claim (sentence 1).

Returning to our sketch of the passage's structure, we might update it as follows.

- Sentence 1: Researchers make a claim about a discovered ship hull.
- Sentence 2: Researchers find evidence for the claim in stories.
- Sentence 3: _____, these stories' description of beeswax blocks offers evidence for the claim.

We've left the blank where the transition word or phrase is needed. Since sentence 3 is a specific case of the general point made in sentence 2 about Siletz Indians' stories, we should expect to select the answer choice that most clearly signals that relationship.

After considering the answer choices, choice B emerges as the best answer. "For example" makes good sense as a transition phrase here because sentence 3 can be read as a particular instance of how the stories referred to in sentence 2 support the researchers' claim in sentence 1.

Choice A is incorrect. "For this reason" implies that the fact that Siletz Indians' stories support the researchers' claim led to the existence of particular stories describing beeswax blocks washing up on shore, which makes no sense in context. Choice C is incorrect because "however" implies that sentence 3 somehow contradicts or undermines sentence 2, which, as we've observed, isn't the case. Choice D is also incorrect. "Likewise" implies that Siletz Indians' stories in general (sentence 2) and their stories about beeswax blocks (sentence 3) are two separate if similar ("like") things supporting the researchers' claim. As we noted earlier, though, sentence 3 elaborates

STRATEGY

When answering Transitions questions, it can be helpful to consider the role of each sentence within the associated passage. In doing so, you can leave out or paraphrase some of the passage's details, as we've done here, in order to focus on the passage's underlying structure.

on sentence 2—we could say that sentence 3 is subordinate to sentence 2—and so it doesn't represent a second distinct example of support for the researchers' claim.

Now let's wrap up this chapter with one more Transitions example. We'll again number elements within the passage for ease of reference in our discussion.

Expression of Ideas Sample Question 5

(1) Seismologists Kaiqing Yuan and Barbara Romanowicz have proposed that the magma fueling Iceland's more than 30 active volcano systems emerges from deep within Earth. (2) The great depths involved—nearly 3,000 km—mark Iceland's volcanoes as extreme outliers; (3) _____ many of Earth's volcanoes are fed by shallow pockets of magma found less than 15 km below the surface.

Which choice completes the text with the most logical transition?

- A) consequently,
- B) in addition,
- C) indeed,
- D) nevertheless,

Let's once again sketch out the relationships among the passage's statements, leaving a blank where the transition word or phrase would appear.

Sentence 1:	Researchers make a claim about Iceland's active volcanoes.
Sentence 2:	The great depth of Iceland's active volcanoes makes them extreme outliers.
Sentence 3:	_____, many active volcanoes are much shallower than Iceland's.

The key task here is to ascertain the role of sentence 3 in relation to sentence 2 and the passage as a whole. Sentence 2 tells us that Iceland's active volcanoes are "extreme outliers" because of their great depth, while sentence 3 informs us that many active volcanoes are much shallower than Iceland's. From this, we realize that sentence 3 amplifies, or strengthens, the point made in sentence 2 by providing more details about shallower active volcanoes.

This might sound like another case in which sentence 3 provides a specific example in support of sentence 2, but that's not quite the case (and "for example" or something similar isn't an answer choice). Rather, sentence 3 essentially restates the point made in sentence 2, only this time with more specifics.

This makes choice C the best answer here. "Indeed" is often used to signal that the statement it introduces builds on what's been written or said previously by reaffirming the underlying truth or accuracy of these

previous statements. In this sense, “indeed” is roughly synonymous with a phrase such as “in reality” or “in truth.” This makes good sense in the context of this passage. Sentence 2 says that Iceland’s active volcanoes are extreme outliers, while sentence 3 says that “in reality” or “in truth,” many other active volcanoes are shallower than Iceland’s.

Choice A is incorrect. “Consequently” signals a cause-effect relationship, but it’s illogical to suggest that many active volcanoes outside of Iceland draw on shallow magma pools *because* Iceland’s volcanoes are extreme outliers. Choice B is also incorrect. “In addition” illogically suggests that sentence 3 describes an additional factor related to the point made in sentence 2, but, as we’ve observed, sentence 3 serves to amplify sentence 2, meaning sentence 3 doesn’t function as a separate, distinct point in the way that “in addition” would imply. Choice D is also incorrect. “Nevertheless” suggests that sentence 3 is true despite what’s presented in sentence 2, but we’ve noted already that this isn’t the case.

Summary: Expression of Ideas Questions

Main purpose: To test your ability to revise passages in order to improve how information and ideas are conveyed

Proportion of the test section: About 20 percent (eight to twelve questions)

Question types

- **Rhetorical Synthesis:** Selectively use and combine provided information and ideas in order to meet specified writerly goals
- **Transitions:** Provide the most logical transition word or phrase in order to link information and ideas in passages

CHAPTER 16

Reading and Writing: Questions—Expression of Ideas Drills

1

While researching a topic, a student has taken the following notes:

- Annie Wu is a prominent American flutist who graduated from the New England Conservatory.
- She has won multiple national flute competitions.
- She is best known for a 2011 YouTube video that has been viewed over two million times.
- The video shows her performing *Three Beats for Beatbox Flute*, an original work by composer Greg Pattillo.
- Wu combines flute playing and beatboxing in the video.

The student wants to emphasize Wu's most well-known achievement.

Which choice most effectively uses relevant information from the notes to accomplish this goal?

- A) Among her many achievements, prominent American flutist Annie Wu graduated from the New England Conservatory and has won multiple national flute competitions.
- B) Annie Wu is best known for a 2011 YouTube video performance of *Three Beats for Beatbox Flute* that has been viewed over two million times.
- C) Composer Greg Pattillo's original work *Three Beats for Beatbox Flute* combines flute playing and beatboxing.
- D) Annie Wu, who has won multiple national flute competitions, has also combined flute playing and beatboxing.

2

While researching a topic, a student has taken the following notes:

- A thermal inversion is a phenomenon where a layer of atmosphere is warmer than the layer beneath it.
- In 2022, a team of researchers studied the presence of thermal inversions in twenty-five gas giants.
- Gas giants are planets largely composed of helium and hydrogen.
- The team found that gas giants featuring a thermal inversion were also likely to contain heat-absorbing metals.
- One explanation for this relationship is that these metals may reside in a planet's upper atmosphere, where their absorbed heat causes an increase in temperature.

The student wants to present the study's findings to an audience already familiar with thermal inversions. Which choice most effectively uses relevant information from the notes to accomplish this goal?

- A) Gas giants were likely to contain heat-absorbing metals when they featured a layer of atmosphere warmer than the layer beneath it, researchers found; this phenomenon is known as a thermal inversion.
- B) The team studied thermal inversions in twenty-five gas giants, which are largely composed of helium and hydrogen.
- C) Researchers found that gas giants featuring a thermal inversion were likely to contain heat-absorbing metals, which may reside in the planets' upper atmospheres.
- D) Heat-absorbing metals may reside in a planet's upper atmosphere.

3

While researching a topic, a student has taken the following notes:

- Muckrakers were journalists who sought to expose corruption in US institutions during the Progressive Era (1897–1920).
- Ida Tarbell was a muckraker who investigated the Standard Oil Company.
- She interviewed Standard Oil Company executives, oil industry workers, and public officials.
- She examined thousands of pages of the company's internal communications, including letters and financial records.
- Her book *The History of the Standard Oil Company* (1904) exposed the company's unfair business practices.

The student wants to emphasize the thoroughness of Ida Tarbell's investigation of the Standard Oil Company. Which choice most effectively uses relevant information from the notes to accomplish this goal?

- A) Ida Tarbell not only interviewed Standard Oil executives, oil industry workers, and public officials but also examined thousands of pages of the company's internal communications.
- B) As part of her investigation of the Standard Oil Company, muckraker Ida Tarbell conducted interviews.
- C) Published in 1904, muckraker Ida Tarbell's book *The History of the Standard Oil Company* exposed the company's unfair business practices.
- D) Ida Tarbell, who investigated the Standard Oil Company, was a muckraker (a journalist who sought to expose corruption in US institutions during the Progressive Era, 1897–1920).

4

While researching a topic, a student has taken the following notes:

- Platinum is a rare and expensive metal.
- It is used as a catalyst for chemical reactions.
- Platinum catalysts typically require a large amount of platinum to be effective.
- Researcher Jianbo Tang and his colleagues created a platinum catalyst that combines platinum with liquid gallium.
- Their catalyst was highly effective and required only trace amounts of platinum (0.0001% of the atoms in the mixture).

The student wants to explain an advantage of the new platinum catalyst developed by Jianbo Tang and his colleagues. Which choice most effectively uses relevant information from the notes to accomplish this goal?

- A) Like other platinum catalysts, the new platinum catalyst requires a particular amount of the metal to be effective.
- B) While still highly effective, the new platinum catalyst requires far less of the rare and expensive metal than do other platinum catalysts.
- C) Platinum is a rare and expensive metal that is used as a catalyst for chemical reactions; however, platinum catalysts typically require a large amount of platinum to be effective.
- D) Researcher Jianbo Tang and his colleagues created a platinum catalyst that combines platinum, a rare and expensive metal, with liquid gallium.

5

A 2017 study of sign language learners tested the role of iconicity—the similarity of a sign to the thing it represents—in language acquisition. The study found that the greater the iconicity of a sign, the more likely it was to have been learned. _____ the correlation between acquisition and iconicity was lower than that between acquisition and another factor studied: sign frequency.

Which choice completes the text with the most logical transition?

- A) In fact,
- B) In other words,
- C) Granted,
- D) As a result,

6

Before the 1847 introduction of the US postage stamp, the cost of postage was usually paid by the recipient of a letter rather than the sender, and recipients were not always able or willing to pay promptly. _____ collecting this fee could be slow and arduous, and heaps of unpaid-for, undeliverable mail piled up in post offices.

Which choice completes the text with the most logical transition?

- A) Regardless,
- B) On the contrary,
- C) Consequently,
- D) For example,

7

The number of dark spots that appear on the Sun, known as sunspots, can vary greatly. For example, there were about 180 sunspots in November 2001. _____ there were only about 2 sunspots in December 2008.

Which choice completes the text with the most logical transition?

- A) In other words,
- B) Similarly,
- C) Therefore,
- D) By comparison,

8

It has long been thought that humans first crossed a land bridge into the Americas approximately 13,000 years ago. _____ based on radiocarbon dating of samples uncovered in Mexico, a research team recently suggested that humans may have arrived more than 30,000 years ago—much earlier than previously thought.

Which choice completes the text with the most logical transition?

- A) As a result,
- B) Similarly,
- C) However,
- D) In conclusion,

Answer Explanations

QUESTION 1

Choice B is the best answer because it most effectively emphasizes Wu's most well-known achievement. This choice focuses directly on her 2011 YouTube video performance of *Three Beats for Beatbox Flute*, which the passage identifies as her best-known accomplishment.

Choices A and D are incorrect because although each choice mentions some of Wu's accomplishments, neither choice mentions her 2011 YouTube video performance of *Three Beats for Beatbox Flute*, which the passage identifies as what Wu is best known for. **Choice C** is incorrect because it doesn't mention Wu at all.

QUESTION 2

Choice C is the best answer because it most effectively presents the study's findings to an audience already familiar with thermal inversions. This choice presents the findings of the study without unnecessarily explaining what thermal inversions are.

Choice A is incorrect because it focuses on explaining what thermal inversions are, which is unnecessary for an audience already familiar with the concept.

Choice B is incorrect because it describes the researchers' approach (methodology), not their findings. **Choice D** is incorrect because although it partially identifies the researchers' findings, it doesn't mention the study itself or link the information to the concept of thermal inversion.

QUESTION 3

Choice A is the best answer because it most effectively emphasizes the thoroughness of Ida Tarbell's investigation of the Standard Oil Company. This choice establishes that for *The History of the Standard Oil Company*, Tarbell interviewed many people from various occupations and reviewed a large amount of documentation.

Choice B is incorrect because it's less successful than choice A in establishing the thoroughness of Tarbell's investigation, as it mentions only that Tarbell conducted interviews (without indicating how many or what sorts of people she interviewed) and leaves out that she also analyzed a large number of documents. **Choice C** is incorrect because it merely describes *The History of the Standard Oil Company* without mentioning how much research went into writing it. **Choice D** is incorrect because it merely describes Tarbell and her work in general terms without mentioning how much research went into writing *The History of the Standard Oil Company*.

QUESTION 4

Choice B is the best answer because it most effectively explains an advantage of the new platinum catalyst developed by Jianbo Tang and his colleagues. This choice establishes such an advantage: the new catalyst is highly effective but requires much less of the rare and expensive metal platinum than do typical platinum catalysts.

Choices A and D are incorrect because each fails to establish an advantage of the new catalyst; choice A stresses a similarity between the new catalyst and other platinum catalysts, while choice D just describes how the new catalyst was created. **Choice C** is incorrect because it doesn't mention the new catalyst at all.

QUESTION 5

Choice C is the best answer because it completes the text with the most logical transition. The sentence with the blank concedes that sign frequency was found to be more important for sign language acquisition than was iconicity, the factor that the previous two sentences discuss. “Granted” means “admittedly” and would logically signal the kind of concession that the passage makes here.

Choice A is incorrect because “in fact” means “in truth” and would serve to emphasize the correctness of the previous sentences by adding new, amplifying details, which is roughly the opposite of the function the sentence with the blank actually performs in the passage. **Choice B** is incorrect because “in other words” would signal that the sentence with the blank restates in simpler language what’s already been said in the passage, whereas the sentence with the blank offers a new piece of information. **Choice D** is incorrect because “as a result” would signal that the sentence with the blank describes the effect of a cause described in the previous sentences, whereas the sentence with a blank offers a concession.

QUESTION 6

Choice C is the best answer because it completes the text with the most logical transition. The sentence with the blank describes two effects—slow fee collection and “heaps” of undeliverable mail—that resulted from the past practice of having the recipient rather than the sender typically pay for postage. “Consequently” means “as a result” and would logically signal this cause-effect relationship.

Choice A is incorrect because “regardless” means “despite everything” and would signal that the sentence with the blank is true in spite of what’s described in the previous sentence, whereas the sentence with the blank logically follows from what’s already been presented in the passage. **Choice B** is incorrect because “on the contrary” means “just the opposite” and would signal that the sentence with the blank contradicts the previous sentence, whereas the sentence with the blank logically follows from what’s already been presented in the passage. **Choice D** is incorrect because “for example” would signal that the sentence with the blank is an example of the situation described in the previous sentence, whereas the sentence with the blank describes a consequence of what’s already been presented in the passage.

QUESTION 7

Choice D is the best answer because it completes the text with the most logical transition. The sentence with the blank completes a comparison between the number of sunspots in November 2001 and in December 2008. “By comparison” would logically signal this comparative relationship.

Choice A is incorrect because “in other words” would signal that the sentence with the blank restates in simpler language what’s already been said in the passage, whereas the sentence with the blank provides a point of contrast. **Choice B** is incorrect because “similarly” would signal that what’s described in the sentence with the blank is comparable to what’s described in the previous sentences, whereas the sentence with the blank provides a point of contrast. **Choice C** is incorrect because “therefore” would signal a cause-effect relationship between the sentence with the blank and the previous sentences, whereas the sentence with the blank provides a point of contrast.

QUESTION 8

Choice C is the best answer because it completes the text with the most logical transition. The sentence with the blank offers evidence that contradicts the previous sentence's assertion that humans first crossed a land bridge into the Americas around 13,000 years ago. "However" means "in spite of that" and would logically signal this contrastive relationship.

Choice A is incorrect because "as a result" would signal a cause-effect relationship between the two sentences in the passage, whereas the sentence with the blank contradicts the previous sentence. **Choice B** is incorrect because "similarly" would signal that the sentence with the blank makes an assertion comparable to the one described in the previous sentence, whereas the sentence with the blank contradicts the previous sentence's assertion. **Choice D** is incorrect because "in conclusion" would signal that the sentence with the blank neatly wraps up the discussion begun in the previous sentence, but this makes no sense in context given that the sentence with the blank contradicts the previous sentence.



CHAPTER 17

Reading and Writing: Questions—Standard English Conventions

Reading and Writing questions in the **Standard English Conventions** content domain are designed to test your ability to edit passages so that they conform to core conventions of Standard English sentence structure, usage, and punctuation. There are two main areas of focus in Standard English Conventions questions: ensuring that sentences are conventionally complete and applying a range of usage and punctuation conventions.

Proportion of the Test Section

About 26 percent of the Reading and Writing section is made up of Standard English Conventions questions. This translates to approximately **eleven to fifteen Standard English Conventions questions** per test form.

Overview of Standard English Conventions Question Types

The Standard English Conventions content domain includes two main question types, each with several subtypes:

- **Boundaries** questions, which test your ability to form conventionally complete sentences
- **Form, Structure, and Sense** questions, which test your ability to edit text to conform to core usage and punctuation conventions

All questions in this content domain (and in the Reading and Writing section in general) are multiple-choice in format. Each question has a single best answer (*key*) and three incorrect answer choices (*distractors*) that represent common errors students make when answering such questions. All the information needed to answer each question is in the passage, so you won't need prior knowledge of the passages' topics to successfully answer associated questions.

“STANDARD ENGLISH” AND “CONVENTIONS”

Before we start examining test questions in this content domain, we need to be clear on a few key concepts.

Standard English is a specific form of formal English that’s typically expected in academic and workplace settings. Because Standard English is widely understood and employed in these settings, it’s useful as a tool of communication. Standard English isn’t the only variety of English, and other forms of English are equally rich and sophisticated, but questions in the Reading and Writing section will only test your understanding of Standard English because of that variety’s connection to college and career readiness.

Conventions in the sense we use the term describe the “rules” of Standard English. We put the word “rules” in quotation marks because these conventions aren’t absolute and unchanging directives but rather agreed-on ways that most users of Standard English adopt to express things. For instance, one expectation of writers using Standard English is that their sentences will express complete thoughts and conform to certain requirements about how sentences are structured and marked with punctuation, such as the inclusion of ending punctuation (periods, question marks, and exclamation points) to denote the conclusion of a sentence. Knowing and applying these conventions makes writing (and, to some extent, speaking) more understandable to others who also know these conventions. In a real sense, the use of conventions is less about conforming to “rules” than it is about enhancing the communicative power of text. To underline that point, all Standard English Conventions questions in the Reading and Writing section appear within realistic contexts, and your task is to edit these contexts to improve their effectiveness.

Because conventions are simply (informal) agreements among writers and speakers about how to express ideas in standard ways, these conventions can—and do—change over time. A commonly cited example of this is the increasing acceptance of the use of the pronouns *they*, *them*, and *theirs* to refer to single individuals. A few language traditionalists still insist that such constructions as “a person . . . they” are incorrect, but the truth is that widely used languages, such as English, grow and evolve over time, and “singular *they*,” as this usage is sometimes referred to, has come into much wider acceptance as part of Standard English. (From a linguistic standpoint, singular *they* actually fills a gap that otherwise exists in Standard English: the lack of a set of gender-neutral singular pronouns.)

The important thing to know here is that we won’t ask you to try to apply conventions that are contested or rapidly changing. This means, to continue our previous example, that we won’t test you on whether “they” is singular or plural in reference to a person whose gender hasn’t been identified (and, to the fullest extent possible, we use the preferred pronouns of individuals mentioned in our passages).

PREPARING TO ANSWER STANDARD ENGLISH CONVENTIONS QUESTIONS

It's beyond the scope of this chapter to teach you the conventions of Standard English. In working through the sample questions in this chapter, we'll also try to keep grammatical terms to a minimum.

Should you, after reading this chapter, feel the need to brush up on some of the conventions, Khan Academy, which hosts Official SAT Practice (khanacademy.org/SAT), has a number of resources dedicated to teaching you about these conventions. Your English language arts textbook (if your class uses one) may also have a grammar, usage, and mechanics section. Numerous other resources, many of them free, are also available in print and digital forms.

Standard English Conventions Questions in Detail

In this section, we'll discuss both Standard English Conventions question types in detail. We'll describe the general purpose of each type so that you understand better how to approach such questions on test day. We'll also discuss the various subtypes of questions in each of these categories. As part of this discussion, we'll walk through some sample questions for each type so that you get a strong sense of the kinds of skills and knowledge required to answer questions of a given type.

Boundaries Questions

Boundaries questions test your ability to apply your understanding of Standard English conventions to ensure that sentences in test passages are conventionally complete. By *conventionally complete*, we mean that the sentences express complete thoughts, are structured in standard ways, and are set off from other sentences by appropriate punctuation.

Questions of this type may ask you to edit sentences in a variety of ways, some of which we'll illustrate below.

Standard English Conventions Sample Question 1

In the novel *Things Fall Apart* by Chinua Achebe, Okonkwo is a leader of Umuofia (a fictional Nigerian clan) and takes pride in his culture's traditions. However, when the arrival of European missionaries brings changes to Umuofia, the novel asks a central question: How _____

Which choice completes the text so that it conforms to the conventions of Standard English?

- A) will Umuofia's traditions be affected?
- B) Umuofia's traditions will be affected?
- C) Umuofia's traditions will be affected.
- D) will Umuofia's traditions be affected.

This question tests your ability to differentiate between two types of sentences: *declarative* sentences, which convey information and end with periods, and *interrogative* sentences, which pose questions and end with question marks.

The passage tells you, via “central question,” that the statement in which the blank appears should be framed as a question (interrogative sentence).

To make this point clearer, consider the statement written in both declarative and interrogative forms.

Declarative: Umuofia's traditions will be affected (in some unspecified way).

Interrogative: How will Umuofia's traditions be affected?

You can observe that not only does the sentence's end punctuation differ between the two versions, but the word order (syntax) also changes.

STRATEGY

Sometimes your “ear” for language may help you recognize obviously incorrect sentence structures. Even if that isn't always the case, it's still a good idea to (silently) read the various answer choices into the sentence by substituting them for the blank.

Since we're told we're looking for a question here, let's consider choices A and B, which are framed as questions. Choice A results in “How will Umuofia's traditions be affected?” This choice matches our analysis and is the best answer because it's the conventionally appropriate choice. Although choice B is similarly presented as a question, it's not in standard interrogative form. “How Umuofia's traditions will be affected?” isn't a conventional way to phrase a question such as this, so choice B can be ruled out as incorrect. Choices C and D are presented as declarative, rather than interrogative, sentences, so we can quickly rule those options out as incorrect.

Standard English Conventions Sample Question 2

Humans were long thought to have begun occupying the Peruvian settlement of Machu Picchu between 1440 and 1450 CE. However, a team led by anthropologist Dr. Richard Burger used accelerator mass spectrometry to uncover evidence that it was occupied _____ 1420 CE, according to Burger, humans were likely inhabiting the area.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A) earlier, which in
- B) earlier, in
- C) earlier. In
- D) earlier in

This question asks you to consider how best to join or separate two statements using words and sometimes punctuation. The passage makes two assertions, one before and one after the blank. To paraphrase:

Researchers discovered that Machu Picchu was occupied earlier (than 1440–1450 CE).

In 1420 CE, humans were inhabiting the area.

Choice C is the best answer because turning these two ideas into two separate, complete sentences is one conventional and effective way to structure the information.

The other answer choices violate convention in differing ways. Choice A would result in “it was occupied earlier, which in 1420 CE . . . humans were likely inhabiting the area,” which is ungrammatical. *Which* is a relative pronoun, but in this case, it wouldn’t precisely refer to anything (i.e., would lack a clear antecedent) in the preceding clause. The relative clause introduced by *which* would also be nonstandard. Choice B is incorrect because it results in a comma splice: two complete sentences joined only by a comma. Choice D is incorrect because it results in a run-on sentence: two complete sentences fused together without an appropriate conjunction and/or punctuation.

Standard English Conventions Sample Question 3

In a 2016 study, Eastern Washington University psychologist Amani El-Alayli found that, among the study participants who experienced frisson (a physiological response akin to goosebumps or getting the chills) while listening to music, there was one personality trait that they scored particularly _____ openness to experience.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A) high on;
- B) high on
- C) high. On
- D) high on:

You may find this question more challenging than the previous two examples because it tests, in part, the appropriate use of colons and semicolons to link ideas. Colons and semicolons aren’t common in informal writing and even in many published texts, but they make frequent appearance in academic (and, to a lesser extent, formal workplace) writing because they offer powerful ways to establish connections between and among ideas. The colon, for example, can be used to introduce a word, phrase, clause, or sentence (or more than one of these) that directly builds or elaborates on the statement made before the colon. This second word, phrase, clause, or sentence can be an example, a definition, a clarification, a restatement, or the like.

NOTE

In published writing, especially literature, sentence fragments (incomplete sentences) are sometimes used for dramatic effect.

Like this, for example,

Standard English Conventions questions, however, won't offer a sentence fragment as the best answer.

A semicolon, on the other hand, can be used to join two (or, less commonly, more) closely related sentences. Its effect is a little different from that of the colon because the semicolon suggests that the two (or more) sentences so connected carry equal weight, whereas the colon implies that what follows that punctuation mark is subordinate to (e.g., exemplifies, defines, clarifies, restates) the statement made before the colon.

Let's turn now to our answer choices for this question. It turns out that choice D, which employs the colon, is the best answer here. The phrase "openness to experience" clarifies what "personality trait" El-Alayli found to be closely associated with study participants who experienced frisson while listening to music.

Choice A is incorrect because a semicolon can't be used in this way to join an independent clause (beginning with "Eastern Washington University") to a phrase ("openness to experience"). As we previously mentioned, conventional use of the semicolon in this manner involves linking two (or more) closely related sentences, not two unequal sentence elements, such as (in this case) an independent clause and a phrase. Choice B is incorrect because some sort of punctuation is required between the independent clause beginning with "Eastern Washington University" and the phrase "openness to experience"; otherwise, the two ideas being expressed run together in a nonstandard and confusing way. Choice C is incorrect because the period after "high" results in two rhetorically unacceptable sentence fragments, one beginning with "in a 2016 study" and the other with "on openness."

Form, Structure, and Sense Questions

The second and final type of Standard English Conventions question consists of those in the **Form, Structure, and Sense** category.

Questions of this type ask you to apply your knowledge of Standard English usage and punctuation conventions to a range of tasks.

Specifically, these questions test

- subject-verb agreement.
- pronoun-antecedent agreement.
- verb finiteness (i.e., contextually appropriate uses of verbs and verbals [gerunds, participles, and infinitives]).
- verb tense and aspect (i.e., contextually proper choice of verb tense and aspect).
- subject-modifier placement (i.e., contextually appropriate placement of modifying elements in sentences).
- genitives and plurals (i.e., distinguishing between plural and possessive forms of nouns and pronouns, and using possessive determiners [e.g., *its*] conventionally).

We provide an example and discussion of each of these question subtypes below.

Standard English Conventions Sample Question 4

Wanda Diaz-Merced, an astrophysicist who is blind, has developed software that can translate astrophysical data into sound. Such tools _____ astrophysicists to detect subtle patterns in data—patterns that may not be evident in graphs and other visual formats.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A) has enabled
- B) enable
- C) is enabling
- D) enables

This question primarily tests **subject-verb agreement**. When we talk about subjects and verbs agreeing, we mean that they have the same number: singular subjects take singular verbs, and plural subjects take plural verbs. Sometimes, as in this example, subject and verb are close together in the sentence, so it's fairly easy to find a match. In more challenging questions, however, the subject and verb may be separated by other words, phrases, and clauses, which makes maintaining the link harder. Sometimes this intervening text may seem to suggest a different number for the verb than is required by the subject itself. In a few cases, a verb may even come before its subject. These questions, though, are still answered the same way: by keeping track of the subject and ensuring that it agrees (matches) in number with the verb.

In this question, choice B is the best answer. The subject of the sentence with the blank is “tools,” which is plural, and “enable” is a plural verb. Choices A, C, and D are incorrect because “has enabled,” “is enabling,” and “enables” are singular verbs that don't agree in number with the plural subject.

Standard English Conventions Sample Question 5

Official measurements of the Mississippi River's length vary: according to the US Geologic Survey, the river is 2,300 miles long, whereas the Environmental Protection Agency records its length as 2,320 miles. This disparity can be explained in part by the fact that rivers such as the Mississippi expand and contract as _____ sediment.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A) one accumulates
- B) they accumulate
- C) it accumulates
- D) we accumulate

This question primarily tests **pronoun-antecedent agreement**. In Standard English, pronouns are expected to agree in number (singular, plural) and person (first person, second person, third person) with the antecedents they refer to. Note that antecedents typically come before the pronouns that rename them, but this isn't always the case.

Choice B is the best answer here. The antecedent for the pronoun that is to complete the text is "rivers," which is plural, and the plural pronoun "they" is the conventionally correct one to use to refer to a plural noun such as "rivers." Choice A is incorrect because "one" is singular, not plural. ("One" in this sense is typically used to refer to a generic person or thing, such as "One really ought to practice for the SAT.") Choice C is incorrect because "it," too, is singular. Choice D is incorrect because although "we" is plural, it doesn't make any sense in context to refer to "rivers" as "we." (Put slightly more technically, "we" is a first person pronoun, and all nouns referring to things take third person pronouns.)

Standard English Conventions Sample Question 6

In 1990, California native and researcher Ellen Ochoa left her position as chief of the Intelligent Systems Technology Branch at a NASA research center _____ the space agency's astronaut training program.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A) to join
- B) is joining
- C) joined
- D) joins

This question primarily tests **verb finiteness**. *Finite verbs* are your average, garden-variety verbs. In a technical sense, verbs are finite when they agree with a subject and take a tense, such as past or present. Finite verbs contrast with a category of words known as *verbals*, which include gerunds (verbs functioning as nouns), participles (verbs functioning as adjectives or that are used as part of compound verbs [e.g., "is walking"]), and infinitives (base forms of verbs, such as "[to] be," that lack the characteristics of agreement or tense and that are used as nouns, adjectives, or adverbs). Conventional sentences require that there be at least one finite verb. Problems with verb finiteness arise when a sentence mistakenly uses a verbal in place of a finite verb, leaving the sentence either incomplete or poorly structured, and when finite verbs are used when verbals are instead called for.

In this case, choice A is the best answer because the context requires the use of the infinitive “to join.” This infinitive signals Ochoa’s intent in leaving one job for another and results in a conventional expression (“Ochoa left . . . to join”). Choices B, C, and D are incorrect because “is joining,” “joined,” and “joins” are finite verbs, whereas the context requires the infinitive “to join.”

Standard English Conventions Sample Question 7

After winning the 1860 presidential election, Abraham Lincoln appointed Edward Bates, Salmon P. Chase, and William H. Seward to his cabinet. Lincoln’s decision was surprising, since each of these men had run against him, but historians have praised it, noting that Lincoln _____ his rivals’ diverse talents to strengthen his administration.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A) will leverage
- B) is leveraging
- C) has leveraged
- D) leveraged

This question primarily tests **verb tense and aspect**. Verb *tense* identifies whether the verb describes something in the past, present, or future. Verb *aspect* can refine tense by adding a specific indication of whether the action took place once, more than once, or progressively over time. The basic guideline here is that verb tense and aspect have to make sense in context. You don’t want a past tense verb to describe present action, for instance, nor do you want to use a verb in, say, the simple past tense (e.g., “ran”) when the action took place over an extended time (e.g., “was running”).

In this question, the best answer is choice D. The context describes how Lincoln made use of his rivals’ talents by appointing several of his former opponents to his cabinet. This event occurred one time in the past (around 1860), so the simple past tense verb “leveraged” makes the most sense.

Choices A, B, and C are incorrect because the simple future tense verb “will leverage,” the present progressive verb “is leveraging,” and the present perfect verb “has leveraged” are inappropriate in this context, which describes a onetime historical event.

Standard English Conventions Sample Question 8

In 2015, a team led by materials scientists Anirudha Sumant and Diana Berman succeeded in reducing the coefficient of friction (COF) between two surfaces to the lowest possible level—superlubricity. A nearly frictionless (and, as its name suggests, extremely slippery) state,

Which choice completes the text so that it conforms to the conventions of Standard English?

- A) reaching superlubricity occurs when two surfaces' COF drops below 0.01.
- B) superlubricity is reached when two surfaces' COF drops below 0.01.
- C) when their COF drops below 0.01, two surfaces reach superlubricity.
- D) two surfaces, when their COF drops below 0.01, reach superlubricity.

This question primarily tests **subject-modifier placement**. Standard English conventions dictate that modifiers be placed as close as possible to the elements of the sentence they modify. Failure to meet this expectation often results in confusing—and sometimes unintentionally amusing—sentences in which the modifier appears to modify something other than what was intended.

In this question, “a nearly frictionless . . . state” is a phrase intended to modify, or describe, superlubricity. To construct this sentence in a conventional way, the modifying phrase should appear as close as possible to the word it modifies. This is accomplished by choice B, which is the best answer.

Choices A, C, and D are incorrect because they illogically result in “a nearly frictionless . . . state” modifying “reaching superlubricity” (choice A) and “two surfaces” (choices C and D). (In slightly more technical terms, “a nearly frictionless . . . state” becomes a dangling modifier under these circumstances.)

Sample Standard English Conventions Question 9

When they were first discovered in Australia in 1798, duck-billed, beaver-tailed platypuses so defied categorization that one scientist assigned them the name *Ornithorhynchus paradoxus*: “paradoxical birdsnout.” The animal, which lays eggs but also nurses _____ young with milk, has since been classified as belonging to the monotremes group.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A) it's
- B) their
- C) they're
- D) its

This question primarily tests appropriate uses of **genitives and plurals**. *Genitives* are a category of (usually) nouns with a range of uses, but for the purposes of answering questions in the Reading and Writing section, genitives are functionally equivalent to possessives—that is, they indicate that something “owns” something else, whether literally (“Alex’s jacket”) or more loosely, as in a trait (“Sofia’s skill”). Essentially, this subtype of Form, Structure, and Sense questions asks you to knowledgeably distinguish among

- possessive nouns.
- plural nouns.
- the possessive determiners (sometimes called *possessive pronouns* and, occasionally, *possessive adjectives*) *my*, *your*, *his*, *her*, *its*, *our*, and *their*.
- the contractions *it’s*, frequently confused with the possessive determiner *its*, and *they’re*, frequently confused with the possessive determiner *their* or the adverb *there*.

The technical distinctions among these categories are fairly easy to grasp, but it’s (its?) their (there? they’re??) application that messes up many people, including some highly educated people and skilled writers. That’s due in part to the fact that *its* and *it’s* as well as *their*, *they’re*, and *there* are indistinguishable to the ear and because *its* and *their* break the typical pattern of possessives in English, which generally take an apostrophe.

One common misapplication of genitives is sometimes described as the “grocer’s apostrophe,” so called because this category of error often appears in signs for the sale of such items as “banana’s” and “apple’s.” The distinctions between *its* (determiner) and *it’s* (contraction) as well as among *their* (determiner), *there* (adverb), and *they’re* (contraction) are just things one has to learn to apply consistently, although asking yourself what function a given frequently confused word performs in a given sentence can help you spot or avoid errors.

Back to the sample question. We want to know which of the answer choices should precede the noun “young,” as in offspring. This is a possessive (genitive) relationship—in a loose sense, the animal (platypus) “owns” its offspring—so the best answer here is the singular possessive determiner *its*, which is choice D.

Choices A and C are incorrect because “it’s” and “they’re” are contractions, not possessive determiners. Choice B is incorrect because although “their” is a possessive determiner, it’s a plural one and therefore doesn’t agree with the singular antecedent “animal.”

STRATEGY

If you’re one of the many people who struggle making these distinctions consistently in your writing, consider what role the word plays in the sentence. The possessive determiner *its*, for example, identifies “ownership” and typically precedes a noun (e.g., “its color is red”), whereas the contraction *it’s* is a shortened form of either “it is” or “it has.” One way to double-check whether *it’s* rather than *its* is appropriate in context is to “read out” the contraction. “It is color is red” makes no sense, so *its* should be used instead. The same can be done with *they’re* and *their*.

Summary: Standard English Conventions

Main purpose: To test your ability to edit passages so that they conform to core conventions of Standard English sentence structure, usage, and punctuation

Proportion of the test section: About 26 percent (11–15 questions)

Question types

- **Boundaries:** Form conventionally complete sentences
- **Form, Structure, and Sense:** Edit text to conform to core usage and punctuation conventions



CHAPTER 18

Reading and Writing: Questions—Standard English Conventions Drills

1

In 1959, marine biologist Dr. Albert Jones founded the Underwater Adventure Seekers, a scuba diving _____ that is the oldest club for Black divers in the United States and that has helped thousands of diving enthusiasts become certified in the field.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A) club
- B) club, and
- C) club—
- D) club,

2

Photographer Ansel Adams's landscape portraits are iconic pieces of American art. However, many of the _____ of landscapes were intended not as art but as marketing; a concessions company at Yosemite National Park had hired Adams to take pictures of the park for restaurant menus and brochures.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A) photographers early photo's
- B) photographers early photos
- C) photographer's early photos
- D) photographer's early photo's

3

The field of geological oceanography owes much to American _____ Marie Tharp, a pioneering oceanographic cartographer whose detailed topographical maps of the ocean floor and its multiple rift valleys helped garner acceptance for the theories of plate tectonics and continental drift. Which choice completes the text so that it conforms to the conventions of Standard English?

- A) geologist
- B) geologist:
- C) geologist;
- D) geologist,

4

In her book *The Woman Warrior: Memoirs of a Girlhood Among Ghosts*, author Maxine Hong Kingston examines themes _____ childhood, womanhood, and Chinese American identity by intertwining autobiography and mythology.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A) of—
- B) of
- C) of:
- D) of,

5

Classical composer Florence Price's 1927 move to Chicago marked a turning point in her career. It was there that Price premiered her First Symphony—a piece that was praised for blending traditional Romantic motifs with aspects of Black folk music—and _____ supportive relationships with other Black artists.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A) developing
- B) developed
- C) having developed
- D) to develop

6

A subseasonal weather forecast attempts to predict weather conditions three to four weeks in _____. Its predictions are therefore more short-term than those of the seasonal forecast, which attempts to predict the weather more than a month in advance.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A) advance and
- B) advance
- C) advance,
- D) advance;

7

In 1881, French chemist Camille Faure redesigned the rechargeable lead-acid battery. Faure's design greatly increased the amount of electricity that the original battery, which the French physicist Gaston Planté _____ fifteen years earlier, could hold.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A) is inventing
- B) will invent
- C) had invented
- D) invents

8

The African Games Co-production Market, one of over 180 annual international conferences supporting video game development, _____ the growth of the African gaming industry by helping start-up studios in Africa find partners.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A) promotes
- B) promote
- C) are promoting
- D) have promoted

Answer Explanations

QUESTION 1

Choice A is the best answer because it completes the text so that it conforms to the conventions of Standard English. The two declarative content clauses ("that is the oldest club for Black divers in the United States," "that has helped thousands of diving enthusiasts become certified in the field") are essential sentence elements used to describe the Underwater Adventure Seekers and therefore shouldn't be separated from the noun "club" by either punctuation or a conjunction.

Choices B, C, and D are incorrect because each separates the noun "club" from the two declarative content clauses with punctuation, a conjunction, or both. **Choice D** is also incorrect because the comma after "club" turns "a scuba diving club" into a nonrestrictive appositive of "Underwater Adventure Seekers," which is inappropriate in this context. The relative clauses that follow ("that is the oldest club for Black divers in the United States," "that has helped thousands of diving enthusiasts become certified in the field") would need to be nonrestrictive themselves and each headed by "which" to make the resultant sentence grammatical.

QUESTION 2

Choice C is the best answer because it completes the text so that it conforms to the conventions of Standard English. The singular possessive noun "photographer's" and the plural noun "photos" are appropriate to indicate that a single photographer (Ansel Adams) took many photographs.

Choices A and B are incorrect because the singular possessive noun "photographer's," not the plural noun "photographers," is needed in this context. **Choices A and D** are incorrect because the plural noun "photos," not the singular possessive noun "photo's," is needed in this context.

QUESTION 3

Choice A is the best answer because it completes the text so that it conforms to the conventions of Standard English. The essential appositive "Marie Tharp" shouldn't be separated by punctuation from the noun phrase ("American geologist") it renames.

Choices B, C, and D are incorrect because each uses punctuation to separate the essential appositive from the noun phrase it renames.

QUESTION 4

Choice B is the best answer because it completes the text so that it conforms to the conventions of Standard English. No punctuation is needed between the preposition "of" and the words "childhood," "womanhood," and "Chinese American identity," each of which functions as an object of the preposition.

Choices A, C, and D are incorrect because each uses punctuation to separate the preposition "of" from its objects.

QUESTION 5

Choice B is the best answer because it completes the text so that it conforms to the conventions of Standard English. This choice establishes parallelism between the two verb phrases ("premiered her First Symphony," "developed supportive relationships with other Black artists") that describe what Price did in Chicago after her move there in 1927.

Choices A, C, and D are incorrect because the present participle "developing," the present perfect participle "having developed," and the infinitive "to develop" aren't parallel in form to the past tense verb "premiered."

QUESTION 6

Choice D is the best answer because it completes the text so that it conforms to the conventions of Standard English. The semicolon after "advance" is used in a conventional way to join the sentence's two closely related independent clauses, the first of which begins with "a subseasonal weather forecast" and the second of which begins with "its predictions."

Choice A is incorrect because it results in a rambling sentence. It's conventional to place a comma after the conjunction "and" when the conjunction is used to join two lengthy independent clauses. Even if that comma were present, however, the sentence would still be awkward, as the conjunction "and" and the conjunctive adverb "therefore" aren't both needed to establish the logical relationship between the two independent clauses. *Choice B* is incorrect because it results in a run-on sentence. *Choice C* is incorrect because it results in a comma splice.

QUESTION 7

Choice C is the best answer because it completes the text so that it conforms to the conventions of Standard English. The past perfect tense verb "had invented" is appropriate in this context to describe a past event that occurred before another event in the past. In this case, Planté invented the rechargeable lead-acid battery fifteen years before Faure improved on the design.

Choices A, B, and D are incorrect because the present progressive tense verb "is inventing," the simple future tense verb "will invent," and the simple present tense verb "invents" are inappropriate to describe a past event that took place before another event in the past.

QUESTION 8

Choice A is the best answer because it completes the text so that it conforms to the conventions of Standard English. The singular verb "promotes" agrees in number with the singular subject "African Games Co-production Market."

Choices B, C, and D are incorrect because "promote," "are promoting," and "have promoted" are plural verbs that don't agree in number with the singular subject "African Games Co-production Market."

PART 3

The Math Section



CHAPTER 19

Math: Overview

The Math section of the digital SAT is, alongside the Reading and Writing section, one of the two main portions of the test. In this test section, you'll answer questions that test your ability to solve problems involving algebra, advanced math, problem-solving and data analysis, and geometry and trigonometry.

In this chapter, we'll provide a high-level overview of the Math section. In the chapters that follow, we'll examine in detail each of the four broad content domains (areas) that compose the section, including the skills and knowledge covered by questions in each of the domains—Algebra, Advanced Math, Problem-Solving and Data Analysis, and Geometry and Trigonometry. Each content domain's overview chapter is followed by a chapter consisting of drill questions (and answer explanations) that you can use for immediate practice.

Three additional chapters round out our discussion of the Math section. The first provides additional details about responding to student-produced response, or SPR, questions in the Math section. As the name suggests, these questions require you to generate your own answers instead of selecting one of four multiple-choice answer options. You'll find samples of questions in this format throughout earlier chapters, but we want to make sure that you know how to properly enter your answers on test day. The second chapter provides an overview of the features of the graphing calculator built into the digital test platform, which you may choose to use on test day, as well as guidance about where to obtain up-to-date information on bringing your own approved calculator. The third chapter consists of a copy of the reference sheet with common math formulas that you have access to during testing.

At a Glance

Table 1 provides an overview of the digital SAT Math section. We then discuss each feature introduced in the table throughout the remainder of the chapter.

PREVIEW

Chapters 20–28 cover the various kinds of questions you'll encounter in the Math section and provide drills that you can use for immediate practice. Chapter 29 offers additional details about the student-produced response (SPR) question format and how to properly enter your answers. Chapter 30 provides an overview of the built-in graphing calculator available to you on the digital test platform as well as guidance about where to obtain up-to-date information on bringing our own approved calculator to test day should you choose to do so. Chapter 31 consists of a copy of the reference sheet available during testing that contains common math formulas.

QUICK TAKE

You'll have a total of **70 minutes** to answer **44 Math questions**, about 75 percent of which are in the multiple-choice format and the remaining roughly 25 percent in the student-produced response (SPR) format. This averages out to **1.59 minutes per question**. These questions will be divided into **two separately timed modules** of 35 minutes each.

Table 1. Math Section Overview.

Feature	SAT Math Section
<i>Timing and Pacing</i>	
Number of questions	1st module: 22 questions
	2nd module: 22 questions
	Total: 44 questions
Time per module	1st module: 35 minutes
	2nd module: 35 minutes
	Total: 70 minutes
Average time per question	1.59 minutes
<i>Score</i>	
Score	Math section score (200–800 scale); one-half of the SAT total score
<i>Contexts</i>	
Proportion of test section	About 30% of questions are in context (“word problems”)
Words per context	A majority of in-context questions have 50 words or fewer
Context areas	Science
	Social studies
	Real-world topics
Graphics (included with select questions)	A wide range of types of informational graphics and geometric figures (e.g., xy-plane graphs, bar graphs, scatterplots)
<i>Questions</i>	
Question format	Four-option multiple-choice, each with a single correct answer (about 75% of section)
	Student-produced response (SPR) (about 25% of section)
Question content domains (categories)	Algebra
	Advanced Math
	Problem-Solving and Data Analysis
	Geometry and Trigonometry
<i>Calculator Use</i>	
A calculator is allowed throughout the Math section. You may use the graphing calculator built into the digital test platform, or you may bring your own approved calculator.	
<i>Reference Sheet</i>	
You’ll have access to a set of common formulas used in math throughout the test section.	
<i>Scratch (Scrap) Paper</i>	
While you’re taking the Math section, you’ll have access to notepaper for performing calculations and the like.	

Discussion

In this section of the chapter, we'll go over each of the features in table 1.

Number of Questions, Timing, and Pacing

Each Math section is divided into two equal-length *modules* of questions. Each of these modules is separately timed, so you'll have 35 minutes to answer the 22 questions in each module. That averages out to 1.59 minutes per question.

Score

You'll receive a Math section score based on your performance on the section. This section score will range from 200 to 800, depending on how well you did. Your scores on the Math and Reading and Writing sections added together will give you your total score for the SAT, which is on a 400–1600 scale.

Contexts

About 30 percent of questions in the Math section are set in context. By *context*, we mean that the question presents a brief scenario that you have to read and analyze in order to solve the associated problem. These questions will ask you to consider topics in social studies, science, or real-world settings. Many of these contexts are quite short—a majority of in-context questions are 50 words or fewer—and all contexts are written to be clear and understandable without the need for prior knowledge of the topics being discussed. In other words, all topic-specific information needed to answer an in-context question correctly is in the question itself, and you won't need to have studied a given science, social studies, or real-world topic in school. These sorts of questions are included in the Math section because they help assess your ability to apply your math skills and knowledge in authentic situations. The remaining roughly 70 percent of questions are “pure” math problems without context.

QUICK TAKE

About 30 percent of Math questions are **set in context**. This context may consist of a **science, social studies, or real-world topic**. However, you won't need prior knowledge of these topics to be able to answer the associated questions correctly, as all relevant information about these topics is included in the contexts themselves.

Graphics

The Math section includes three main varieties of graphics with select questions. One category consists of informational graphics that display data. These include the same sorts of tables, line graphs, and bar graphs that you'll be presented with in the Reading and Writing section as well as other informational graphics types commonly encountered in math classes, such as scatterplots, dot plots, and histograms. The second category consists of graphs of functions in the *xy*-plane, while the third consists of geometric figures, such as triangles. You'll be expected to make skillful use of these graphics and the information they contain and represent to answer associated questions.

QUICK TAKE

Select Math questions are accompanied by one or more graphics. These may be **informational graphics** (data displays), **graphs of functions in the *xy*-plane**, or **geometric figures**, such as triangles.

QUICK TAKE

About 75 percent of Math questions are in the **multiple-choice format**, with each question having a single correct answer among four answer options. The other **roughly 25 percent** are in the **student-produced response format**. These questions require you to generate and enter your own answers.

Questions

Two question formats are used in the Math section. About 75 percent of the questions are in the multiple-choice format. Each of these questions has four answer choices (options), one (and only one) of which is the correct answer, or *key*. The three incorrect answer choices—which are sometimes called *distractors*—represent common errors that students often make in answering math questions, either in how they approach the question or how they calculate the answer. Your task in these questions is to use your math skills and knowledge to figure out which of the four choices is the correct answer.

The rest of the Math section's questions—about 25 percent of the total—use the student-produced response, or SPR, format. As the name suggests, these questions don't include answer choices to pick from. Instead, you'll have to find and enter your answers on your own. These questions are intended to find out whether you can successfully apply your math skills and knowledge without the structure of a set of options to choose from. We devote chapter 29 to discussing how to properly enter answers to questions in the SPR format, and examples of SPR questions are found throughout other chapters. For now, you just need to know that about one-quarter of the Math section's questions will require you to come up with and enter your own answer. Note that some SPR questions may have more than one possible correct answer, but you'll still enter only one answer per question.

All Math questions are independent of each other, or *discrete*. This means that there are no question sets built around a common source, such as a passage or graphic. This is good news for you because it means you can approach each question separately, give your best answer, and move on to the next question.

Each Math question, whether multiple-choice or student-produced response, primarily tests a single skill or element of knowledge (e.g., your ability to solve linear equations in one variable). The knowledge and skills tested by Math questions fall into four broad categories, which we call *content domains*. Each of these domains represents a major area of focus in math. Table 2 offers an overview of the four content domains in the Math section, including each domain's "big idea," or main focus; the skills and knowledge each domain covers; and the approximate proportion of the Math section devoted to questions in each domain. Subsequent chapters go into detail about each of the domains in turn and provide opportunities for immediate practice.

QUICK TAKE

Each Math question falls into one of four categories, or **content domains**: Algebra, Advanced Math, Problem-Solving and Data Analysis, and Geometry and Trigonometry. Each question primarily tests one skill or knowledge element (e.g., solving linear equations in one variable).

Table 2. Math Section Content Domains.

Math Section Content Domain	The Big Idea	Skills and Knowledge Tested	Proportion of Test Section
Algebra	Questions in this domain test your understanding of linear relationships.	- Linear equations in one variable - Linear equations in two variables - Linear functions - Systems of two linear equations in two variables - Linear inequalities in one or two variables	13–15 questions (about 35%)
Advanced Math	Questions in this domain test your understanding of nonlinear relationships.	- Equivalent expressions - Nonlinear equations in one variable and systems of equations in two variables - Nonlinear functions	13–15 questions (about 35%)
Problem-Solving and Data Analysis	Questions in this domain test your understanding of proportional relationships, percentages, and probability as well as your ability to use data to analyze and solve problems.	- Ratios, rates, proportional relationships, and units - Percentages - One-variable data: distributions and measures of center and spread - Two-variable data: models and scatterplots - Probability and conditional probability - Inference from sample statistics and margin of error - Evaluating statistical claims: observational studies and experiments	5–7 questions (about 15%)
Geometry and Trigonometry	Questions in this domain test your understanding of concepts central to geometry and trigonometry.	- Area and volume - Lines, angles, and triangles - Right triangles and trigonometry - Circles	5–7 questions (about 15%)

IMPORTANT: UNSCORED QUESTIONS

Four Math questions on each test form—two different questions per module—won't count toward your section score. These questions are ones that College Board is studying for potential use on future tests. Your answers to these questions won't impact your section score in any way. You won't be able to tell these questions apart from the ones in the section that do get scored, but the small number of these unscored questions shouldn't affect your practice or test day performance. Just do your best on each question.

Questions from all four domains appear in each test module (i.e., in each of the two separately timed portions of the Math section). Questions in each module are ordered by difficulty from easiest to hardest, regardless of the content domain the question belongs to, so you may find it advantageous to answer questions in the order in which they appear.

QUICK TAKE

An **approved calculator** is allowed for all Math questions.

Calculator Use

You're allowed to use a calculator on all questions on the Math section. The digital test platform itself has a built-in graphing calculator that you can use if you wish, or you may instead use your own approved calculator. We recommend basing this choice on which tool you feel more comfortable with. We'll talk more about calculator options in chapter 30. The key thing for the moment is that you may use a calculator with all Math questions if you wish, although, as we'll talk about later, using a calculator won't always be the most efficient way to answer questions.

Reference Sheet

While you're taking the Math section, you'll have ready access to a reference sheet that includes a set of common math formulas that you can make use of anytime you want during the test. A copy of this reference sheet appears in chapter 31.

Scratch (Scrap) Paper

Even though you'll take the digital SAT on a laptop, desktop computer, or other device, you'll still have access on test day to notepaper for performing calculations, taking notes, and the like.

Summary: Math Section

Let's quickly recap the basics of the digital SAT's Math section:

Timing and pacing

- 70 minutes to answer 44 questions divided into two separately timed modules
- An average of 1.59 minutes to answer each question

Score

- A Math section score on a 200–800 scale (one-half of the total score for the digital SAT)

Contexts

- About 30 percent of Math questions set in context
- Majority of in-context questions have 50 words or fewer
- Contexts drawn from topics in science, social studies, and real-world settings

Graphics

- Informational graphics (data displays), graphs of functions in the xy-plane, or geometric figures included with select questions

Questions

- A mix of four-option multiple-choice questions (about 75 percent of the section total) and student-produced response (SPR) questions (about 25 percent of the section total)
 - ♦ Multiple-choice questions: A single correct answer (*key*)
 - ♦ SPR questions: May have more than one correct answer, although only a single answer is entered
- All questions independent of each other (i.e., no question sets)
- Content domains of Algebra, Advanced Math, Problem-Solving and Data Analysis, and Geometry and Trigonometry
- Ordered from easiest to hardest within each module
- 4 unscored questions (2 unique questions per module)

Calculators

- Allowed for all Math questions
- May choose between built-in graphing calculator and an approved personal calculator

Reference sheet

- A set of common formulas available anytime while taking the Math section

Scratch (scrap) paper

- Notepaper available during the Math section for making calculations, taking notes, and the like

CHAPTER 20

Math: Questions— Introduction

The following chapters discuss the kinds of questions you'll encounter in the Math section and give you opportunities to immediately practice what you've learned.

Recall that the Math section has questions in four broad categories, which we refer to as *content domains*. Each content domain has a “big idea,” or common focus, tying all the questions in the domain together at a conceptual level. Table 1 summarizes the section's four content domains and the focus of their respective questions.

Table 1. Math Section Content Domains.

Math Content Domain	The Big Idea
Algebra (chapters 21 and 22)	Testing your understanding of linear relationships
Advanced Math (chapters 23 and 24)	Testing your understanding of nonlinear relationships
Problem-Solving and Data Analysis (chapters 25 and 26)	Testing your understanding of proportional relationships, percentages, and probability as well as your ability to use data to analyze and solve problems
Geometry and Trigonometry (chapters 27 and 28)	Testing your understanding of concepts central to geometry and trigonometry

Each of the following pairs of chapters will

- provide a broad overview of a particular content domain, including the “big idea” (focus) of the domain;
- indicate about how many questions you can expect to encounter from that domain on test day;
- identify the key types of questions included in the domain;
- offer a range of sample questions and answer explanations; and
- give you an opportunity to immediately practice by answering questions from that domain and reading the associated answer explanations.

CHAPTER 21

Math: Questions— Algebra

Algebra questions in the Math section focus on the mastery of linear equations and inequalities, linear functions, and systems of linear equations. The ability to interpret, create, use, and solve problems using linear representations and to make connections between different representations of linear relationships is essential for success in college and careers.

Across the Math section, Algebra questions vary significantly in form and appearance. They may be straightforward fluency exercises or pose challenges of strategy or understanding, such as interpreting the relationship between graphical and algebraic representations or solving as a process of reasoning. You'll be required to demonstrate both procedural skill and a deep understanding of concepts.

The questions in the Algebra content domain include both multiple-choice questions and student-produced response (SPR) questions.

Let's explore the content, skills, and knowledge assessed by Algebra questions.

Linear Equations, Linear Inequalities, and Linear Functions in Context

When you use algebra to analyze and solve a problem in real life, a key step is to represent the context of the problem algebraically. To do this, you may need to define one or more variables that represent quantities in the context. Then you may need to write one or more expressions, equations, inequalities, or functions that represent the relationships described in the context. For some algebra questions, you may need to rewrite an equation or interpret a given algebraic representation. For other questions, once you write an equation that represents the context, you then need to solve that equation. Then you may need to interpret the solution to the equation in terms of the context. Questions in the Math section may assess your ability to accomplish any or all of these steps.

QUICK TAKE

Questions in the Math section require you to demonstrate deep understanding of several core algebra topics, namely linear equations and inequalities, linear functions, and systems of linear equations. These topics are fundamental to the learning and work often required in college and careers.

REMINDER

Multiple-choice questions have four answer choices and one (and only one) correct answer, or *key*. Questions in the student-produced response (SPR) format require you to generate and enter your own answer.

STRATEGY

Many Algebra questions such as this one will require you to perform the following steps:

1. Define one or more variables that represent quantities in the question.
2. Write one or more equations, expressions, inequalities, or functions that represent the relationships described in the question.
3. Solve the equation.
4. Interpret the solution in terms of what the question is asking.

Ample practice with each of these steps will help you develop the math skills and knowledge needed to successfully answer questions in the Algebra content domain.

Algebra Sample Question 1

In 2014, County X had 783 miles of paved roads. Starting in 2015, the county has been building 8 miles of new paved roads each year. At this rate, how many miles of paved road will County X have in 2030? (Assume that no paved roads go out of service.)

The first step in answering this question is to decide what variable or variables you need to define. Since the number of miles paved depends on the year, we can define a variable to represent the year. The number of years after 2014 can be represented using the variable n . Then, since the question says that County X had 783 miles of paved road in 2014 and has been building 8 miles of new paved roads each year, the expression $783 + 8n$ gives the number of miles of paved roads in County X in the year that is n years after 2014. The year 2030 is $2030 - 2014 = 16$ years after 2014; thus, the year 2030 corresponds to $n = 16$. Hence, to find the number of miles of paved roads in County X in 2030, substitute 16 for n in the expression $783 + 8n$, giving $783 + 8(16)$. This is equivalent to $783 + 128$, or 911. Therefore, at the given rate of building, County X will have 911 miles of paved roads in 2030.

Note that this example provides no answer choices. It's in the student-produced response format mentioned briefly above. To respond to this question, you would enter your answer in the digital test delivery platform. Directions for entering your answers to student-produced response questions are presented in chapter 29.

Note that the same context could have generated different sorts of questions, as in samples 2 and 3, below.

Algebra Sample Question 2

In 2014, County X had 783 miles of paved roads. Starting in 2015, the county has been building 8 miles of new paved roads each year. At this rate, which of the following functions f gives the number of miles of paved road there will be in County X n years after 2014? (Assume that no paved roads go out of service.)

- A) $f(n) = 8 + 783n$
- B) $f(n) = 2,014 + 783n$
- C) $f(n) = 738 + 8n$
- D) $f(n) = 2,014 + 8n$

This question already defines the variable and asks you to identify a function that describes the context. The discussion for sample 1 shows that the correct answer is choice C.

REMINDER

There are several different ways you may be tested on the same underlying algebra concepts.

Practicing a variety of questions with different contexts is a good way to ensure you'll be ready for the questions you'll come across in the Math section.

Algebra Sample Question 3

In 2014, County X had 783 miles of paved roads. Starting in 2015, the county has been building 8 miles of new paved roads each year. At this rate, in which year will County X first have at least 1,000 miles of paved roads? (Assume that no paved roads go out of service.)

In this question, you must create and solve an inequality. As in sample 1, let n be the number of years after 2014. Then the expression $783 + 8n$ gives the number of miles of paved roads in County X n years after 2014. The question is asking when there will first be at least 1,000 miles of paved roads in County X. This condition can be represented by the inequality $783 + 8n \geq 1,000$. To find the year in which there will first be at least 1,000 miles of paved roads, you solve this inequality for n . Subtracting 783 from each side of $783 + 8n \geq 1,000$ gives $8n \geq 217$. Then dividing each side of $8n \geq 217$ by 8 gives $n \geq 27.125$. Note that an important part of relating the inequality $783 + 8n \geq 1,000$ back to the context is to notice that n is counting calendar years, and so the value of n must be an integer. The least value of n that satisfies $783 + 8n \geq 1,000$ is 27.125, but the year $2014 + 27.125 = 2041.125$ does not make sense as an answer, and in 2041, there would be only $783 + 8(27) = 999$ miles of paved roads in the county. Therefore, the variable n needs to be rounded up to the next integer, and so the least possible value of n is 28. Therefore, the year that County X will first have at least 1,000 miles of paved roads is 28 years after 2014, which is 2042.

In sample 1, once the variable n was defined, you needed to find an expression that represents the number of miles of paved road in terms of n . In other questions, creating the correct expression, equation, or function may require a more insightful understanding of the context.

Algebra Sample Question 4

To edit a manuscript, Miguel charges \$50 for the first 2 hours and \$20 per hour after the first 2 hours. Which of the following expresses the amount, C , in dollars, Miguel charges if it takes him x hours to edit a manuscript, where $x > 2$?

- A) $C = 20x$
- B) $C = 20x + 10$
- C) $C = 20x + 50$
- D) $C = 20x + 90$

The question defines the variables C and x and asks you to express C in terms of x . To create the correct equation, you must note that since the \$50 that Miguel charges pays for his first 2 hours of editing, he charges \$20 per hour only *after* the first 2 hours. Thus, if it takes x hours for Miguel to edit a manuscript, he charges \$50 for the first 2 hours and \$20 per hour for the remaining time, which is $x - 2$ hours. Thus, his total charge, C , in dollars, can be written as $C = 50 + 20(x - 2)$,

STRATEGY

Solving an equation or inequality is often only part of the problem-solving process. You'll also need to interpret the solution in the context of the question, so be sure to remind yourself of the question's context and the meaning of the variables you solved for before selecting your answer.

STRATEGY

When the solution you arrive at doesn't match any of the answer choices provided in a given multiple-choice question, consider whether expanding, simplifying, or rearranging your solution will cause it to match an answer choice. Sometimes this extra step is needed to arrive at the correct answer.

where $x > 2$. This doesn't match any of the provided answer choices. But when you apply the distributive property to the right-hand side of $C = 50 + 20(x - 2)$, you get $C = 50 + 20x - 40$, or $C = 20x + 10$, which is choice B.

As with samples 1 to 3, different questions could have been asked about this context. For example, you could be asked to find how long it took Miguel to edit a manuscript if he charged \$370.

In some questions in the Math section, you'll be given a function that represents a context and be asked to find the value of the output of the function given an input or, as in sample 5, below, the value of the input that corresponds to a given output.

Algebra Sample Question 5

A builder uses the function g defined by $g(x) = 110x + 10,000$ to estimate the cost $g(x)$, in dollars, to build a one-story home of planned floor area of x square feet in Stillwater. If the builder estimates that the cost to build a certain one-story home in Stillwater is \$142,000, what is the planned floor area, in square feet, of the home?

This question asks you to find the value of the input of a function when you're given the value of the output and the equation of the function. The estimated cost of the home, in dollars, is the output of the function g for a one-story home of planned floor area of x square feet. That is, the output of the function, $g(x)$, is 142,000, and you need to find the value of the input x that gives an output of 142,000. To do this, substitute 142,000 for $g(x)$ in the equation that defines g : $142,000 = 110x + 10,000$. Now solve for x : First, subtract 10,000 from each side of the equation $142,000 = 110x + 10,000$, which gives $132,000 = 110x$. Then, divide each side of $132,000 = 110x$ by 110, which gives $1,200 = x$. Therefore, a one-story home with an estimated cost of \$142,000 to build in Stillwater has a planned floor area of 1,200 square feet.

Systems of Linear Equations and Inequalities in Context

You may need to define more than one variable and create more than one equation or inequality to represent a context and answer a question. Questions on the Math section may require you to create and solve a system of equations or create a system of inequalities.

Algebra Sample Question 6

Maizah bought pants and a briefcase at a department store. The sum of the prices of the pants and the briefcase before sales tax was \$130.00. There was no sales tax on the pants and a 9% sales tax on the briefcase. The total Maizah paid, including the sales tax, was \$136.75. What was the price, in dollars, of the pants?

To answer the question, you first need to define the variables. The question discusses the prices of pants and a briefcase and asks you to find the price of the pants. So it's appropriate to let P be the price, in dollars, of the pants and to let B be the price, in dollars, of the briefcase. Since the sum of the prices before sales tax was \$130.00, the equation $P + B = 130$ represents the sum of the prices. A sales tax of 9% was added to the price of the briefcase. Since 9% is equal to 0.09, the price of the briefcase with tax was $B + 0.09B = 1.09B$. There was no sales tax on the pants, and the total Maizah paid, including tax, was \$136.75, so the equation $P + 1.09B = 136.75$ represents the total, in dollars, Maizah paid.

Now you need to solve the system

$$\begin{aligned} P + B &= 130 \\ P + 1.09B &= 136.75 \end{aligned}$$

Subtracting the left- and right-hand sides of the first equation from the corresponding sides of the second equation gives you $(P + 1.09B) - (P + B) = 136.75 - 130$, which can be rewritten as $0.09B = 6.75$. Now you can divide each side of the equation

$0.09B = 6.75$ by 0.09. This gives you $B = \frac{6.75}{0.09}$, or $B = 75$. Thus, the price, in dollars, of the briefcase is 75. The question asks for the price, in dollars, of the pants, which is P . You can substitute 75 for B in the equation $P + B = 130$, which gives you $P + 75 = 130$, or $P = 130 - 75$, or $P = 55$, so the price of the pants is \$55.

Algebra Sample Question 7

Each morning, John jogs at 6 miles per hour and rides a bike at 12 miles per hour. His goal is to jog and ride his bike a total of at least 9 miles in no more than 1 hour. If John jogs j miles and rides his bike b miles, which of the following systems of inequalities represents John's goal?

A) $\begin{cases} \frac{j}{6} + \frac{b}{12} \leq 1 \\ j + b \geq 9 \end{cases}$

B) $\begin{cases} \frac{j}{6} + \frac{b}{12} \geq 1 \\ j + b \leq 9 \end{cases}$

C) $\begin{cases} 6j + 12b \geq 9 \\ j + b \leq 1 \end{cases}$

D) $\begin{cases} 6j + 12b \leq 1 \\ j + b \geq 9 \end{cases}$

It's given that John jogs j miles and rides his bike b miles. It's also given that his goal is to jog and ride his bike a total of at least 9 miles. This goal is represented by the inequality $j + b \geq 9$. This eliminates choices B and C.

STRATEGY

You can use either of two approaches—combination or substitution—when solving a system of linear equations. One may get you to the answer more quickly than the other, depending on the equations you're working with and what you're solving for. Practice using both approaches to give you greater flexibility on test day.

STRATEGY

While this question may seem complex, as it involves numerous steps, solving it calls on the same underlying principles outlined earlier: defining variables, creating equations to represent relationships, solving equations, and interpreting the solution.

STRATEGY

In sample 7, the answer choices each contain two parts. Use this to your advantage by tackling one part at a time and eliminating answers that don't work.

STRATEGY

You should be able to quickly rearrange equations such as $\text{rate} \times \text{time} = \text{distance}$ by solving for any of the variables. Sample 7 requires you to solve the equation for time.

Since $\text{rate} \times \text{time} = \text{distance}$, it follows that $\text{time} = \frac{\text{distance}}{\text{rate}}$. John jogs j miles at 6 miles per hour, so the time he jogs is equal to $\frac{j \text{ miles}}{6 \text{ miles/hour}} = \frac{j}{6}$ hours. Similarly, since John rides his bike b miles at 12 miles per hour, the time he rides his bike is $\frac{b}{12}$ hours. Thus, John's goal to complete his jog and his bike ride in no more than 1 hour can be represented by the inequality $\frac{j}{6} + \frac{b}{12} \leq 1$. The system $j + b \geq 9$ and $\frac{j}{6} + \frac{b}{12} \leq 1$ is choice A.

Fluency in Solving Linear Equations, Linear Inequalities, and Systems of Linear Equations

Creating linear equations, linear inequalities, and systems of linear equations that represent a context are key skills for success in college and careers. It's also essential to be able to fluently solve these linear equations, linear inequalities, and systems of linear equations. Some of the Algebra questions in the Math section may also present equations, inequalities, or systems without a context and directly assess your fluency in solving them.

All such fluency questions in the Math section permit the use of a calculator, and some of them test your ability to solve equations, inequalities, and systems of equations. Even though a calculator is permitted for all Math section questions, you may be able to answer certain questions more quickly without using a calculator, such as in sample 9, below. Part of what the Math section assesses is your ability to decide whether using a calculator to answer a question offers efficiency or whether it's more efficient to solve by hand. Sample 8 is an example of a question that could be solved either by hand or by using a graphing calculator.

Algebra Sample Question 8

$$3\left(\frac{1}{2} - x\right) = \frac{3}{5} + 15x$$

What is the solution to the given equation?

Using the distributive property to rewrite the left-hand side of the equation gives $\frac{3}{2} - 3x = \frac{3}{5} + 15x$. Adding $3x$ to both sides of this equation and then subtracting $\frac{3}{5}$ from both sides of this equation gives $\frac{3}{2} - \frac{3}{5} = 18x$. The equation may be easier to solve if it's transformed

into an equation without fractions; to do this, multiply each side of $\frac{3}{2} - \frac{3}{5} = 18x$ by 10, which is the least common multiple of the denominators 2 and 5. This gives $\frac{30}{2} - \frac{30}{5} = 180x$, which can be rewritten as $15 - 6 = 180x$, or $9 = 180x$. Dividing both sides of this equation by 180 gives $x = \frac{1}{20}$.

Alternatively, each side of the given equation can be set equal to y and entered in a graphing calculator. Then the problem can be solved by finding the intersection point of the two lines that each represent one side of the given equation. You would enter $y = 3\left(\frac{1}{2} - x\right)$ and $y = \frac{3}{5} + 15x$ in the graphing interface and draw the graphs. The intersection point is $(0.05, 1.35)$, which means $x = 0.05$, which is equivalent to $x = \frac{1}{20}$. Note that .05 and $1/20$ are examples of ways to enter a correct answer.

Algebra Sample Question 9

$$-2(3x + 2.4) = -3(3x + 2.4)$$

What is the solution to the given equation?

You could solve this in the same way as sample 8, by multiplying everything out and simplifying, or by graphing each side of the equation. But the structure of the equation reveals that -2 times a quantity, $3x + 2.4$, is equal to -3 times the same quantity. This is only possible if the quantity $3x + 2.4$ is equal to zero. Thus, $3x + 2.4 = 0$, or $3x = -2.4$. Therefore, the solution is $x = -0.8$.

Algebra Sample Question 10

$$-2x = 4y + 6$$

$$2(2y + 3) = 3x - 5$$

What is the solution (x, y) to the given system of equations?

- A) $(1, 2)$
- B) $(1, -2)$
- C) $(-1, -1)$
- D) $(-1, 1)$

This is an example of a system you can solve more efficiently by substitution than by using the elimination method demonstrated in sample 6. Since $-2x = 4y + 6$, it follows that $-x = 2y + 3$. Now you can substitute $-x$ for $2y + 3$ in the second equation. This gives you $2(-x) = 3x - 5$, which simplifies to $5x = 5$, or $x = 1$. Substituting 1 for x in the first equation gives you $-2 = 4y + 6$, which simplifies to $4y = -8$, or $y = -2$. Therefore, the solution to the system is $(1, -2)$.

STRATEGY

While a calculator is permitted on all questions in the Math section, it's important to not rely too much on the tool. Some questions, such as sample 9, can be solved more efficiently without using a calculator. Your ability to choose when to use and when not to use a calculator is one of the things the Math section assesses, so be sure to practice this.

STRATEGY

In sample 6, the elimination method yields an efficient solution to the question. In sample 10, the substitution method turns out to be an efficient approach. These examples illustrate the benefits of knowing both approaches and thinking critically about which approach may be more efficient for a given question.

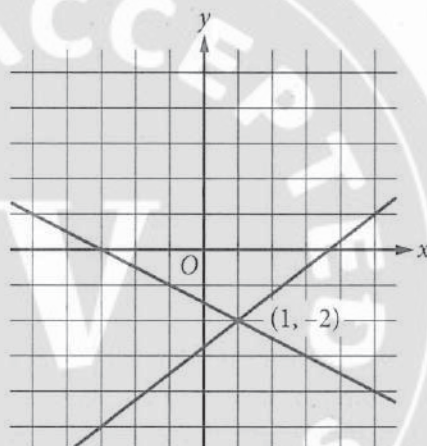
In the preceding examples, you found a unique solution to linear equations and to systems of two linear equations in two variables. But not all such equations and systems have solutions, and some have infinitely many solutions. Some questions in the Math section assess your ability to determine whether an equation or a system of linear equations has one solution, no solutions, or infinitely many solutions.

The Relationships among Linear Equations, Lines in the Coordinate Plane, and the Contexts They Describe

STRATEGY

Graphing systems of two linear equations is another effective approach to solving them. Practice arranging linear equations into $y = mx + b$ form and graphing them in the coordinate plane.

A system of two linear equations in two variables can be solved by graphing the lines in the coordinate plane. For example, you can graph the equations of the system in the xy -plane in sample 10, above.



The point of intersection gives the solution to the system.

If the equations in a system of two linear equations in two variables are graphed, each graph will be a line. There are three possibilities:

1. The lines intersect at one point. In this case, the system has a unique solution.
2. The lines are parallel. In this case, the system has no solution.
3. The lines are identical. In this case, every point on the line is a solution, and so the system has infinitely many solutions.

One way that the second and third cases can be identified is by rewriting the equations of the system in slope-intercept form, $y = mx + b$, where m is the slope of the line and b is the y -coordinate of the y -intercept of the line when the system is graphed in the xy -plane. If the lines have the same slope and different y -coordinates of the y -intercepts, the lines are parallel; if both the slope and the y -coordinate of the y -intercept are the same, the lines are identical.

How are the second and third cases represented algebraically? Samples 11 and 12 answer this question.

Algebra Sample Question 11

$$2y + 6x = 3$$

$$y + 3x = 2$$

How many solutions (x, y) does the given system of equations have?

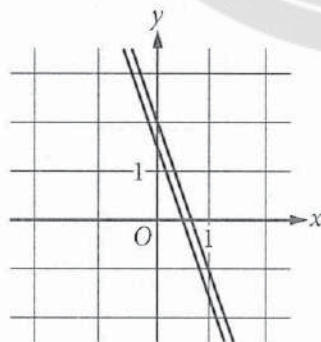
- A) Zero
- B) Exactly one
- C) Exactly two
- D) Infinitely many

To rewrite the second equation in slope-intercept form, subtract $3x$ from both sides of the equation, which gives $y = -3x + 2$. To rewrite the first equation in slope-intercept form, subtract $6x$ from both sides of the equation, which gives $2y = -6x + 3$. Next, divide both sides of this equation by 2, which gives $y = -3x + \frac{3}{2}$. Now that the two equations are written in slope-intercept form, you can identify that the slope of each line is -3 and that the y -coordinates of the y -intercepts are $\frac{3}{2}$ and 2 .

Since the slopes are the same and the y -coordinates of the y -intercepts are different, the graphs of the lines will show distinct parallel lines. Because parallel lines never intersect, the system of equations has zero solutions.

Alternatively, if you multiply each side of $y + 3x = 2$ by 2, you get $2y + 6x = 4$. Then subtracting each side of $2y + 6x = 3$ from the corresponding side of $2y + 6x = 4$ gives $0 = 1$. This is a false statement. Therefore, the system has zero solutions (x, y) .

You could also use graphing technology to graph the two equations. The graphs are parallel lines, so there are no points of intersection, and the system of equations has zero solutions.



Algebra Sample Question 12

$$3s - 2t = a$$

$$-15s + bt = -7$$

In the given system of equations, a and b are constants. If the system has infinitely many solutions, what is the value of a ?

STRATEGY

When the graphs of a system of two linear equations are distinct parallel lines, as in sample 11, the system has zero solutions. If the question states that a system of two linear equations has an infinite number of solutions, as in sample 12, the equations must be equivalent.

STRATEGY

The equations in the system in sample 11 are in a form that allows you to quickly find both the x -intercept and the y -intercept of the graph of the equation. For example, the graph of $y + 3x = 2$ has an x -intercept of $(\frac{2}{3}, 0)$ because if $y = 0$, then $3x = 2$ and $x = \frac{2}{3}$. Similarly, the graph has a y -intercept of $(0, 2)$ because if $x = 0$, then $y = 2$.

If a system of two linear equations in two variables has infinitely many solutions, the two equations in the system must be equivalent. Since the two equations are presented in the same form, the second equation must be equal to the first equation multiplied by a constant. Since the coefficient of s in the second equation is -5 times the coefficient of s in the first equation, multiply each side of the first equation by -5 . This gives you the system

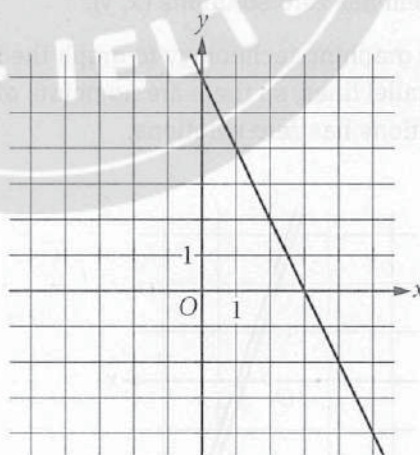
$$\begin{aligned} -15s + 10t &= -5a \\ -15s + bt &= -7 \end{aligned}$$

Since these two equations are equivalent and have the same coefficient of s , the coefficients of t and the constants on the right-hand side must also be the same. Thus, $b = 10$ and $-5a = -7$. Therefore, the value of a is $\frac{7}{5}$.

There may also be questions in the Math section that assess your knowledge of the relationship between the algebraic and the geometric representations of a line—that is, between an equation of a line and its graph. The key concepts are

- If the slopes of line ℓ and line k are each defined (that is, if neither line is a vertical line), then
 - ♦ Line ℓ and line k are parallel if and only if they have the same slope.
 - ♦ Line ℓ and line k are perpendicular if and only if the product of their slopes is -1 .

Algebra Sample Question 13



The graph of line k is shown in the xy -plane. Which of the following is an equation of a line that is perpendicular to line k ?

- A) $y = -2x + 1$
- B) $y = -\frac{1}{2}x + 2$
- C) $y = \frac{1}{2}x + 3$
- D) $y = 2x + 4$

Note that the graph of line k passes through the points $(0, 6)$ and $(3, 0)$. Thus, the slope of line k is $\frac{0-6}{3-0} = -2$. Since the product of the slopes of perpendicular lines is -1 , a line that is perpendicular to line k will have slope $\frac{1}{2}$. All the choices are in slope-intercept form, and so the coefficient of x is the slope of the line represented by the equation. Therefore, choice C, $y = \frac{1}{2}x + 3$, is an equation of a line with slope $\frac{1}{2}$, and thus this line is perpendicular to line k .

As we've noted, some contexts can be described with a linear equation. The graph of a linear equation is a line. A nonvertical line has geometric properties such as its slope and its y -intercept. These geometric properties can often be interpreted in terms of the context. The Math section may have questions that assess your ability to make these interpretations. For example, reconsider the contexts in samples 1 to 3. You created a linear function, $f(n) = 783 + 8n$, that describes the number of miles of paved road County X will have n years after 2014. This equation can be graphed in the coordinate plane, with n on the horizontal axis and $f(n)$ on the vertical axis. The points of this graph lie on a line with slope 8 and y -intercept of $(0, 783)$. The slope, 8, gives the number of miles of new paved roads added each year, and the y -intercept gives the number of miles of paved roads in 2014, the year that corresponds to $n = 0$.

Algebra Sample Question 14

A voter registration drive was held in Town Y. The number of voters, V , registered T days after the drive began can be estimated by the equation $V = 3,450 + 65T$. What is the best interpretation of the number 65 in this context?

- A) The estimated number of registered voters at the beginning of the registration drive
- B) The estimated number of registered voters at the end of the registration drive
- C) The total estimated number of voters registered during the drive
- D) The estimated number of voters registered each day during the drive

The correct answer is choice D. For each day that passes, it's the next day of the registration drive, and so T increases by 1. In the given equation, when T , the number of days after the drive began, increases by 1, V , the estimated number of voters registered, becomes $V = 3,450 + 65(T + 1)$, or $V = 3,450 + 65T + 65$. That is, the estimated number of voters registered increases by 65 for each day of the drive. Therefore, 65 is the estimated number of voters registered each day during the drive.

You should note that choice A describes the number 3,450, and the numbers described by choices B and C can be found only if you know how many days the registration drive lasted; this information isn't given in the question.

REMINDER

The Math section will further assess your understanding of linear equations by, for instance, asking you to select a linear equation that describes a given graph, select a graph that describes a given linear equation, or determine how a graph may be affected by a change in its equation.

STRATEGY

Sample 13 requires a strong understanding of slope as well as the ability to calculate slope: slope is equal to rise over run, or the change in the y -value divided by the change in the x -value. Parallel lines have slopes that are equal. Perpendicular lines have slopes whose product is -1 .

Summary: Algebra Questions

Main purpose: To assess your understanding of linear relationships

Proportion of the test section: About 35 percent (approximately thirteen to fifteen questions)

Question types

- Linear equations in one variable
- Linear equations in two variables
- Linear functions
- Systems of two linear equations in two variables
- Linear inequalities in one or two variables



CHAPTER 22

Math: Questions— Algebra Drills

1

$$3a + 4b = 25$$

A shipping company charged a customer \$25 to ship some small boxes and some large boxes. The given equation represents the relationship between a , the number of small boxes, and b , the number of large boxes, the customer had shipped. If the customer had 3 small boxes shipped, how many large boxes were shipped?

- A) 3
- B) 4
- C) 5
- D) 6

2

$$ax = 5$$

In the given equation, a is a constant. For which of the following values of a will the equation have no solution?

- A) 0
- B) 1
- C) 5
- D) 10

3

Tom scored 85, 78, and 98 on his first three exams in history class. Solving which inequality gives all the possible scores, G , that Tom could get on his fourth exam that will result in a mean score on all four exams of at least 90?

- A) $90 - (85 + 78 + 98) \leq 4G$
- B) $4G + 85 + 78 + 98 \geq 360$
- C) $\frac{(G + 85 + 78 + 98)}{4} \geq 90$
- D) $\frac{(85 + 78 + 98)}{4} \geq 90 - 4G$

4

If $3(3x + 5) = 2x - 8$, what is the value of x ?

- A) $-\frac{23}{7}$
- B) $-\frac{15}{7}$
- C) $-\frac{13}{7}$
- D) $\frac{7}{11}$

5

On Monday, Jao walked a total of 11,400 steps. On Tuesday, Jao has a goal to walk at least 1,500 more steps than he did on Monday. What is the least number of steps Jao could walk on Tuesday to meet his goal?

6

$$x - 3y = 7$$

$$3y = 9$$

If (x, y) is the solution to the given system of equations, what is the value of x ?

- A) -2
- B) 10
- C) 16
- D) 34

7

$$P = 1.20x + 5.00$$

The given equation gives the total monthly price P , in dollars, for using an online gaming service. The total monthly price for the online service consists of a flat monthly fee and a charge for each game played during a month. Of the following, which is the best interpretation of the value of x in this context?

- A) The number of games played during a month
- B) The charge, in dollars, for playing x games
- C) The flat monthly fee, in dollars, for the gaming service
- D) The number of months the gaming service was used

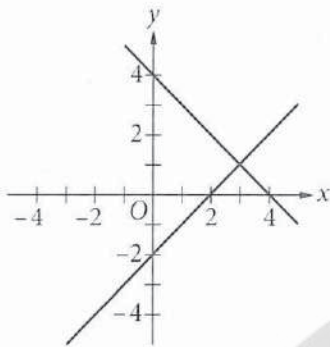
8

$$x + y = 4$$

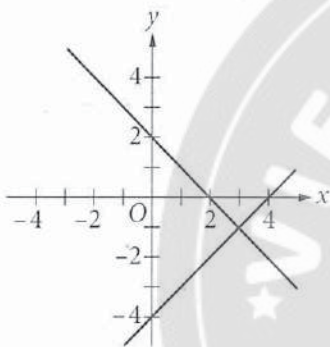
$$x - y = 2$$

Which of the following is the graph in the xy -plane of the given system of equations?

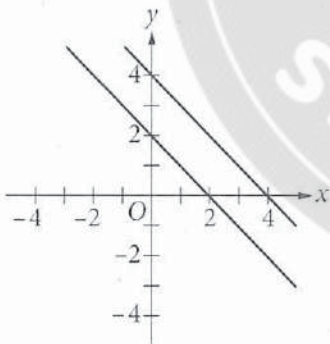
A)



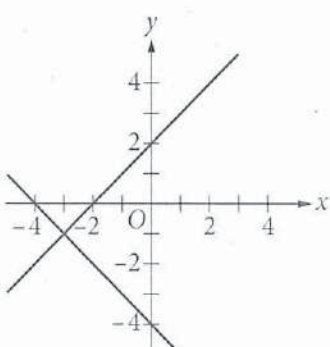
B)



C)



D)



9

Nayya burns 5 kilocalories per minute running on a treadmill and 10 kilocalories per minute pedaling on a stationary bike. Which of the following equations represents the total number of kilocalories, T , Nayya has burned after running on the treadmill for 50 minutes and pedaling on the stationary bike for m minutes?

- A) $T = 15m + 50$
- B) $T = 50m + 50$
- C) $T = 5m + 500$
- D) $T = 10m + 250$

10

$$f(x) = \frac{(x+7)}{4}$$

For the function f defined as shown, what is the value of $f(9) - f(1)$?

- A) 1
- B) 2
- C) $\frac{1}{4}$
- D) $\frac{9}{4}$

11

In the xy -plane, line l contains the points (2, 6) and (8, 10). Which of the following is an equation of line l ?

- A) $y = \frac{2}{3}x + \frac{14}{3}$
- B) $y = \frac{3}{2}x - 2$
- C) $y = 2x + 6$
- D) $y = 8x + 10$

12

During a month, Morgan ran r miles at 5 miles per hour and biked b miles at 10 miles per hour. She ran and biked a total of 200 miles that month, and she biked for twice as many hours as she ran. What is the total number of miles that Morgan biked during the month?

- A) 80
- B) 100
- C) 120
- D) 160

13

The equation $c = \frac{5}{4}x + 406$ gives the total cost c , in dollars, to produce a quantity of x units. If the quantity of units produced increases by 39 units, what is the corresponding increase in the total cost, in dollars?

14

$$kx - 3y = 4$$

$$4x - 5y = 7$$

In the given system of equations, k is a constant and x and y are variables. For what value of k will the system of equations have no solution?

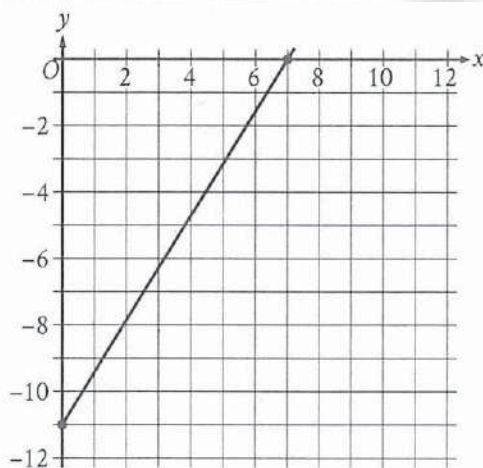
- A) $\frac{12}{5}$
- B) $\frac{16}{7}$
- C) $-\frac{16}{7}$
- D) $-\frac{12}{5}$

15

x	y
-10	66
-5	45
5	3
10	-18

The table shows four values of x and their corresponding values of y . There is a linear relationship between x and y . If an equation representing this relationship is written in the form $Ax + 5y = C$, where A and C are constants, what is the value of C ?

16



The point $(3, d)$ lies on the line shown. What is the value of d ?

Answer Explanations

QUESTION 1

Choice B is correct. It's given that a represents the number of small boxes and b represents the number of large boxes the customer had shipped. If the customer had 3 small boxes shipped, then $a = 3$. Substituting 3 for a in the equation $3a + 4b = 25$ yields $3(3) + 4b = 25$, or $9 + 4b = 25$. Subtracting 9 from both sides of this equation yields $4b = 16$. Dividing both sides of this equation by 4 yields $b = 4$. Therefore, the customer had 4 large boxes shipped.

Choice A is incorrect. If the customer had 3 small boxes and 3 large boxes shipped, then the company would have charged the customer \$21, not \$25.

Choice C is incorrect. If the customer had 3 small boxes and 5 large boxes shipped, then the company would have charged the customer \$29, not \$25.

Choice D is incorrect. If the customer had 3 small boxes and 6 large boxes shipped, then the company would have charged the customer \$33, not \$25.

QUESTION 2

Choice A is correct. An equation has no solution when there is no value of x that produces a true statement. To solve the given equation for x , both sides of the equation can be divided by a , yielding $x = \frac{5}{a}$. Since 5 can be divided by any number except 0, it follows that the value of a can't be 0. Therefore, if the value of a is 0, the given equation will have no solution.

Alternate approach: Substituting 0 for a in the given equation yields $0x = 5$, or $0 = 5$, which isn't a true statement. Therefore, when $a = 0$, there is no solution to the given equation.

Choices B, C, and D are incorrect. These values of a yield exactly one solution rather than no solution.

QUESTION 3

Choice C is correct. The mean of the four scores (G , 85, 78, and 98) can be expressed as $\frac{G + 85 + 78 + 98}{4}$. Thus, an inequality that represents a mean score on all four exams of at least 90 can be written as $\frac{G + 85 + 78 + 98}{4} \geq 90$.

Choices A, B, and D are incorrect and may result from conceptual or calculation errors.

QUESTION 4

Choice A is correct. Applying the distributive property of multiplication on the left-hand side of the given equation yields $3(3x) + 3(5) = 2x - 8$, or $9x + 15 = 2x - 8$. Subtracting $2x$ from both sides of this equation yields $7x + 15 = -8$. Subtracting 15 from both sides of this equation yields $7x = -23$. Dividing both sides of this equation by 7 yields $x = -\frac{23}{7}$.

Choices B, C, and D are incorrect and may result from conceptual or calculation errors.

QUESTION 5

The correct answer is 12,900. It's given that Jao walked a total of 11,400 steps on Monday. It's also given that Jao has a goal to walk at least 1,500 more steps on Tuesday than he did on Monday. Let x represent the possible number of steps Jao could walk on Tuesday to meet his goal. This situation can be represented by the inequality $x \geq 11,400 + 1,500$, or $x \geq 12,900$. Thus, the least number of steps Jao could walk on Tuesday to meet his goal is 12,900.

QUESTION 6

Choice C is correct. From the given system of equations, adding the second equation, $3y = 9$, to the first equation, $x - 3y = 7$, yields $x - 3y + 3y = 7 + 9$, or $x = 16$.

Choices A, B, and D are incorrect and may result from conceptual or calculation errors.

QUESTION 7

Choice A is correct. It's given that the total monthly price P , in dollars, is the sum of a flat monthly fee and a charge for each game played during a month. A charge for each game played implies the multiplication of the rate, in dollars per game, and the number of games played. Therefore, \$1.20 must be the charge per game, and x must be the number of games played during a month.

Choice B is incorrect. The charge, in dollars, for playing x games is $1.20x$.

Choice C is incorrect. The flat monthly fee, in dollars, for the gaming service is \$5.00. **Choice D** is incorrect. No information has been provided about the number of months the gaming service was used.

QUESTION 8

Choice A is correct. Each of the equations in the given system can be rewritten in the form $y = mx + b$, where m represents the slope and $(0, b)$ represents the y -intercept of the line. The first equation in the given system can be rewritten as $y = -x + 4$. It follows that this line has a slope of -1 and a y -intercept at $(0, 4)$. The second equation in the given system can be rewritten as $y = x - 2$. It follows that this line has a slope of 1 and a y -intercept at $(0, -2)$. Of the given choices, only the graph in choice A represents these lines.

Choices B and D are incorrect. These systems of equations represent lines with y -intercepts at $(0, 2)$ and $(0, -4)$ rather than $(0, 4)$ and $(0, -2)$. **Choice C** is incorrect. This system of equations represents lines with y -intercepts at $(0, 2)$ and $(0, 4)$ rather than $(0, 4)$ and $(0, -2)$.

QUESTION 9

Choice D is correct. It's given that Nayya burns 5 kilocalories per minute running on a treadmill. It follows that she will burn $5(50)$, or 250, kilocalories running on the treadmill for 50 minutes. It's also given that Nayya burns 10 kilocalories per minute pedaling on a stationary bike. It follows that she will burn $10m$ kilocalories pedaling on the stationary bike for m minutes. The sum of the kilocalories burned after running on the treadmill for 50 minutes, 250, and the kilocalories burned after pedaling on the stationary bike for m minutes, $10m$, gives the total number of kilocalories, T , Nayya burned. Therefore, $T = 250 + 10m$, or $T = 10m + 250$.

Choice A is incorrect. This equation represents the total number of kilocalories Nayya has burned if she burns 15, not 10, kilocalories per minute pedaling on a stationary bike and ran on the treadmill for 10, not 50, minutes. *Choice B* is incorrect. This equation represents the total number of kilocalories Nayya has burned if she burns 50, not 10, kilocalories per minute pedaling on a stationary bike and ran on the treadmill for 10, not 50, minutes. *Choice C* is incorrect. This equation represents the total number of kilocalories Nayya has burned if she burns 5, not 10, kilocalories per minute pedaling on a stationary bike and ran on the treadmill for 100, not 50, minutes.

QUESTION 10

Choice B is correct. The value of $f(9)$ represents the value of $f(x)$ when $x = 9$. Substituting 9 for x in the given function yields $f(9) = \frac{(9+7)}{4}$, which can be rewritten as $f(9) = \frac{16}{4}$, or $f(9) = 4$. Similarly, the value of $f(1)$ represents the value of $f(x)$ when $x = 1$. Substituting 1 for x in the given function yields $f(1) = \frac{(1+7)}{4}$, which can be rewritten as $f(1) = \frac{8}{4}$, or $f(1) = 2$. Therefore, $f(9) - f(1) = 4 - 2$, which is equivalent to 2.

Choices A, C, and D are incorrect and may result from conceptual or calculation errors.

QUESTION 11

Choice A is correct. The equation of a line in the xy -plane can be written as $y = mx + b$, where m represents the slope and b represents the y -coordinate of the y -intercept of the line. Given two points on a line, (x_1, y_1) and (x_2, y_2) , the slope of the line can be calculated as $m = \frac{y_2 - y_1}{x_2 - x_1}$. Substituting $(2, 6)$ for (x_1, y_1) and $(8, 10)$ for (x_2, y_2) in the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$ yields $m = \frac{10 - 6}{8 - 2}$, which is equivalent to $m = \frac{4}{6}$, or $m = \frac{2}{3}$. Substituting $\frac{2}{3}$ for m in the equation $y = mx + b$ yields $y = \frac{2}{3}x + b$. Since it's given that line l contains the point $(2, 6)$, substituting 2 for x and 6 for y in $y = \frac{2}{3}x + b$ yields $6 = \frac{2}{3}(2) + b$, or $6 = \frac{4}{3} + b$. Subtracting $\frac{4}{3}$ from both sides of this equation yields $b = \frac{14}{3}$. Substituting $\frac{14}{3}$ for b in the equation $y = \frac{2}{3}x + b$ gives $y = \frac{2}{3}x + \frac{14}{3}$.

Choice B is incorrect and may result from reversing x and y when calculating the slope. *Choice C* is incorrect and may result from using the x - and y -coordinates of the first given point as the slope and constant, respectively, in the equation. *Choice D* is incorrect and may result from using the x - and y -coordinates of the second given point as the slope and constant, respectively, in the equation.

QUESTION 12

Choice D is correct. The number of hours Morgan spent running or biking can be calculated by dividing the distance, in miles, she traveled during that activity by her speed, in miles per hour, for that activity. It's given that Morgan ran r miles at 5 miles per hour and biked b miles at 10 miles per hour. It follows that the number of hours she ran can be represented by the expression $\frac{r}{5}$ and the number of hours she biked can be represented by the expression $\frac{b}{10}$. It's given that she biked for twice as many hours as she ran, so this can be represented by the equation $\frac{b}{10} = 2\left(\frac{r}{5}\right)$, which can be rewritten as $5b = 20r$, or $b = 4r$. It's also given that she ran and biked a total of 200 miles. This can be represented by the

equation $r + b = 200$. Substituting $4r$ for b in this equation yields $r + 4r = 200$, or $5r = 200$. Dividing both sides of this equation by 5 yields $r = 40$. Determining the number of miles she biked, b , can be found by substituting 40 for r in the equation $r + b = 200$, which yields $40 + b = 200$. Subtracting 40 from both sides of this equation yields $b = 160$. Therefore, Morgan biked a total of 160 miles during the month.

Choices A, B, and C are incorrect because they don't satisfy the condition of Morgan biking for twice as many hours as she ran. If she had biked 80 miles (choice A), then she would have run 120 miles, which means she would have biked for 8 hours and run for 24 hours. If she had biked 100 miles (choice B), then she would have run 100 miles, which means she would have biked for 10 hours and run for 20 hours. If she had biked 120 miles (choice C), then she would have run 80 miles, which means she would have biked for 12 hours and run for 16 hours.

QUESTION 13

The correct answer is 48.75. It's given that the equation $c = \frac{5}{4}x + 406$ gives the total cost c , in dollars, to produce a quantity of x units. Since $c = \frac{5}{4}x + 406$ is a linear equation with a positive rate of change, $\frac{5}{4}$, it follows that the value of c increases by $\frac{5}{4}$ for every increase in x by 1. Therefore, if the value of x increases by 39, the value of c increases by $\frac{5}{4}(39)$, or $\frac{195}{4}$, which is equivalent to 48.75. Thus, if the quantity of units produced increases by 39, the corresponding increase in the total cost, in dollars, is 48.75. Note that 48.75 and $195/4$ are examples of ways to enter a correct answer.

QUESTION 14

Choice A is correct. If a system of two linear equations has no solution, then the lines represented by the equations in the coordinate plane are parallel and distinct. The equation $kx - 3y = 4$ can be rewritten as $y = \frac{k}{3}x - \frac{4}{3}$, where $\frac{k}{3}$ is the slope and $(0, -\frac{4}{3})$ is the y -intercept of the line. The equation $4x - 5y = 7$ can be rewritten as $y = \frac{4}{5}x - \frac{7}{5}$, where $\frac{4}{5}$ is the slope and $(0, -\frac{7}{5})$ is the y -intercept of the line. If the two lines are parallel, then the lines have the same slope and different y -intercepts. Therefore, $\frac{k}{3} = \frac{4}{5}$, or $k = \frac{12}{5}$.

Choices B, C, and D are incorrect. These values of k each yield a system of equations that has exactly one solution rather than no solution.

QUESTION 15

The correct answer is 120. It's given that there is a linear relationship between x and y and that the table shows four values of x and their corresponding values of y . Based on the given table, when $x = -5$, the corresponding value of y is 45 and when $x = 5$, the corresponding value of y is 3. Substituting -5 for x and 45 for y in the equation $Ax + 5y = C$ yields $A(-5) + 5(45) = C$, or $-5A + 225 = C$. Substituting 5 for x and 3 for y in the equation $Ax + 5y = C$ yields $A(5) + 5(3) = C$, or $5A + 15 = C$. Since $-5A + 225 = C$ and $5A + 15 = C$, it follows that $-5A + 225 = 5A + 15$. Subtracting $5A$ and 225 from both sides of this equation yields $-10A = -210$. Dividing both sides of this equation by -10 yields $A = 21$. Substituting 21 for A in the equation $-5A + 225 = C$ yields $-5(21) + 225 = C$, which is equivalent to $-105 + 225 = C$, or $C = 120$. Therefore, if an equation representing this relationship is written in the form $Ax + 5y = C$, where A and C are constants, the value of C is 120.

QUESTION 16

The correct answer is $-\frac{44}{7}$. It's given from the graph that the points $(7, 0)$ and $(0, -11)$ lie on the line shown in the xy -plane. For two points on a line, (x_1, y_1) and (x_2, y_2) , the slope, m , of the line can be calculated using the slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$. Substituting $(7, 0)$ for (x_1, y_1) and $(0, -11)$ for (x_2, y_2) in this formula, the slope of the line can be calculated as $m = \frac{-11 - 0}{0 - 7}$, or $m = \frac{11}{7}$. It's also given that the point $(3, d)$ lies on the line. Substituting $(7, 0)$ for (x_1, y_1) , $(3, d)$ for (x_2, y_2) , and $\frac{11}{7}$ for m in the slope formula yields $\frac{11}{7} = \frac{d - 0}{3 - 7}$, or $\frac{11}{7} = \frac{d}{-4}$. Multiplying both sides of this equation by -4 yields $(-4)\left(\frac{11}{7}\right) = d$, or $d = -\frac{44}{7}$. Thus, the value of d is $-\frac{44}{7}$. Note that $-44/7$, -6.285 , and -6.286 are examples of ways to enter a correct answer.



CHAPTER 23

Math: Questions— Advanced Math

Advanced Math questions in the Math section address topics that are especially important for students to master before studying higher-level math concepts. Chief among these topics is the understanding of the structure of expressions and the ability to analyze, manipulate, and rewrite these expressions. Questions in this content domain also include ones involving reasoning with more complex equations and interpreting and building functions.

Algebra questions focus on the mastery of linear equations and inequalities, systems of linear equations, and linear functions. By contrast, Advanced Math questions focus on the ability to work with and analyze more complex equations. Advanced Math questions may require you to demonstrate procedural skill in adding, subtracting, and multiplying polynomials and in factoring polynomials. You may also be required to work with expressions involving integer and rational exponents, radicals, or fractions with a variable in the denominator. Questions in this domain may ask you to solve quadratic, radical, rational, polynomial, or absolute value equations. They may also ask you to solve a system consisting of a linear equation and a nonlinear equation. You may also be required to manipulate an equation in several variables to isolate a quantity of interest.

Some questions in Advanced Math may ask you to build a quadratic or an exponential function or an equation that describes a context or to interpret the function, the graph of the function, or the solution to an equation in terms of the context.

Advanced Math questions may assess your ability to recognize structure. Expressions and equations that appear complex may use repeated terms or repeated expressions. By noticing these patterns, the complexity of a problem can be reduced. Structure may be used to factor or otherwise rewrite an expression, to solve a quadratic or other equation, or to draw conclusions about the context represented by an expression, equation, or function. You may be asked to identify or derive the form of an expression, equation, or function that reveals information about the expression, equation, or function or the context it represents.

Advanced Math questions may also assess your understanding of functions and their graphs. A question may require you to demonstrate your understanding of function notation, including interpreting an expression in which the argument of a function is an expression rather than a variable. The questions may assess your understanding of how the algebraic properties of a function relate to the geometric characteristics of its graph.

Advanced Math questions include both multiple-choice questions and student-produced response questions. Although you may always use a calculator in the Math section, you must decide whether using it is an effective strategy for a given question.

Let's consider the content, skills, and knowledge assessed by Advanced Math questions.

Operations with Polynomials and Rewriting Expressions

Questions in the Math section may assess your ability to add, subtract, and multiply polynomials.

Advanced Math Sample Question 1

$$x^3 + 6x^2 + 7x - 6 = (x^2 + bx - 2)(x + 3)$$

In the given equation, b is a constant. If the equation is true for all values of x , what is the value of b ?

- A) 2
- B) 3
- C) 7
- D) 9

The given equation shows two polynomials that are equivalent. To find the value of b , use the distributive property to rewrite the polynomial on the right-hand side of the given equation and then combine like terms so that the polynomial on the right-hand side is in the same form as the polynomial on the left-hand side.

$$\begin{aligned} x^3 + 6x^2 + 7x - 6 &= (x^2 + bx - 2)(x + 3) \\ &= (x^3 + bx^2 - 2x) + (3x^2 + 3bx - 6) \\ &= x^3 + (3 + b)x^2 + (3b - 2)x - 6 \end{aligned}$$

Since the two polynomials are equal for all values of x , the coefficient of matching powers of x should be the same. Therefore, comparing the coefficients of $x^3 + 6x^2 + 7x - 6$ and $x^3 + (3 + b)x^2 + (3b - 2)x - 6$ reveals that $3 + b = 6$ and $3b - 2 = 7$. Solving either of these equations gives $b = 3$, which is choice B.

STRATEGY

The skills and knowledge tested in Advanced Math questions build on the knowledge and skills tested by Algebra questions. Develop proficiency with the tested Algebra skills and knowledge before tackling Advanced Math questions.

Questions may also ask you to use structure to rewrite expressions. For example, the expression may be in the form of a common pattern, such as a difference of squares, where using the difference-of-squares structure allows for efficient rewriting of the expression as the product of two binomials, as shown in sample 2.

Advanced Math Sample Question 2

Which of the following is equivalent to $16s^4 - 4t^2$?

- A) $4(s^2 - t)(4s^2 + t)$
- B) $4(4s^2 - t)(s^2 + t)$
- C) $4(2s^2 - t)(2s^2 + t)$
- D) $(8s^2 - 2t)(8s^2 + 2t)$

A close examination reveals that the given expression follows the difference of two perfect squares pattern, $x^2 - y^2$, which factors as the product $(x - y)(x + y)$. The expression $16s^4 - 4t^2$ is also the difference of two squares: $16s^4 - 4t^2 = (4s^2)^2 - (2t)^2$. Therefore, it can be factored as $(4s^2)^2 - (2t)^2 = (4s^2 - 2t)(4s^2 + 2t)$. This expression can be rewritten as $(4s^2 - 2t)(4s^2 + 2t) = 2(2s^2 - t)(2)(2s^2 + t)$, or $4(2s^2 - t)(2s^2 + t)$, which is choice C.

Alternatively, a 4 could be factored out of the given expression: $4(4s^4 - t^2)$. The expression inside the parentheses is a difference of two squares. Therefore, it can be further factored as $4(2s^2 - t)(2s^2 + t)$.

Advanced Math Sample Question 3

Which expression is equivalent to $xy^2 + 2xy^2 + 3xy$?

- A) $2xy^2 + 3xy$
- B) $3xy^2 + 3xy$
- C) $6xy^4$
- D) $6xy^5$

There are three terms in the expression, the first two of which are like terms. The like terms can be added together by adding their coefficients: $xy^2 + 2xy^2 + 3xy = (xy^2 + 2xy^2) + 3xy$, which is equivalent to $3xy^2 + 3xy$. Therefore, choice B is correct.

Quadratic Functions and Equations

Advanced Math questions may require you to build a quadratic function or an equation to represent a context.

STRATEGY

Advanced Math questions require a high comfort level working with quadratic equations and expressions, including multiplying polynomials and factoring. Recognizing classic quadratic patterns such as $x^2 - y^2 = (x - y)(x + y)$ can also improve your speed and accuracy.

Advanced Math Sample Question 4

A car is traveling at x feet per second. The driver sees a red light ahead, and after 1.5 seconds reaction time, the driver applies the brake. After the brake is applied, the car takes $\frac{x}{24}$ seconds to stop, during which time the average speed of the car is $\frac{x}{2}$ feet per second. If the car travels 165 feet from the time the driver saw the red light to the time it comes to a complete stop, which of the following equations can be used to find the value of x ?

- A) $x^2 + 48x - 3,960 = 0$
- B) $x^2 + 48x - 7,920 = 0$
- C) $x^2 + 72x - 3,960 = 0$
- D) $x^2 + 72x - 7,920 = 0$

STRATEGY

Sample 4 requires careful translation of an in-context problem into an algebraic equation. It pays to be deliberate and methodical when translating such problems in the Math section into equations.

STRATEGY

Questions in the Advanced Math domain may ask you to solve a quadratic equation. Practice using the various methods (below) until you're comfortable with all of them.

1. Factoring
2. Completing the square
3. Using the quadratic formula
4. Graphing using technology
5. Using structure

Be prepared to identify which strategy may be the most effective for solving a given quadratic equation.

During the 1.5-second reaction time, the car is still traveling at x feet per second, so it travels a total of $1.5x$ feet. The average speed of the car during the $\frac{x}{24}$ -second braking interval is $\frac{x}{2}$ feet per second, so over this interval the car travels $\left(\frac{x}{2}\right)\left(\frac{x}{24}\right) = \frac{x^2}{48}$ feet. Since the total distance the car travels from the time the driver saw the red light to the time the car comes to a complete stop is 165 feet, you have the equation $\frac{x^2}{48} + 1.5x = 165$. This quadratic equation can be rewritten in standard form by subtracting 165 from each side and then multiplying each side by 48, giving $x^2 + 72x - 7,920 = 0$, which is choice D.

Some questions in the Advanced Math domain may ask you to solve a quadratic equation. You must determine an appropriate procedure: factoring, completing the square, using the quadratic formula, graphing using technology, or using structure. It may be more efficient to solve a quadratic equation if you can make use of one of the following facts:

- The sum of the distinct solutions of $x^2 + bx + c = 0$ is $-b$.
- The product of the distinct solutions of $x^2 + bx + c = 0$ is c .

Each of these facts can be observed from examining the factored form of a quadratic. If r and s are distinct solutions of $x^2 + bx + c = 0$, then $x^2 + bx + c = (x - r)(x - s)$. Thus, $b = -(r + s)$ and $c = (-r)(-s) = rs$.

Note: To use either of these facts, the coefficient of x^2 must be equal to 1.

Advanced Math Sample Question 5

What are the solutions to the equation $x^2 - 3 = x$?

- A) $\frac{-1 \pm \sqrt{11}}{2}$
- B) $\frac{-1 \pm \sqrt{13}}{2}$
- C) $\frac{1 \pm \sqrt{11}}{2}$
- D) $\frac{1 \pm \sqrt{13}}{2}$

The equation can be solved by using the quadratic formula or by completing the square. Let's use the quadratic formula. First, subtract x from each side of $x^2 - 3 = x$ to put the equation into standard form: $x^2 - x - 3 = 0$. The quadratic formula states that the solutions, x , of the equation $ax^2 + bx + c = 0$ are $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. For the equation $x^2 - x - 3 = 0$, $a = 1$, $b = -1$, and $c = -3$. Substituting these values into the quadratic formula gives $x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-3)}}{2(1)}$. This equation can be rewritten as $x = \frac{1 \pm \sqrt{1 - (-12)}}{2}$, or $x = \frac{1 \pm \sqrt{13}}{2}$, which is choice D.

REMINDER

The quadratic formula states that the solutions x of the equation $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Advanced Math Sample Question 6

If $x > 0$ and $2x^2 + 3x - 2 = 0$, what is the value of x ?

The left-hand side of the given equation can be factored: $2x^2 + 3x - 2 = (2x - 1)(x + 2)$. Therefore, the given equation can be rewritten as $(2x - 1)(x + 2) = 0$, which implies that either $2x - 1 = 0$, which gives $x = \frac{1}{2}$, or $x + 2 = 0$, which gives $x = -2$. Since $x > 0$, the value of x is $\frac{1}{2}$.

STRATEGY

Pay close attention to all the details in each question. In sample 6, x can equal $\frac{1}{2}$ or -2 , but since the question states that $x > 0$, the value of x must be $\frac{1}{2}$.

Advanced Math Sample Question 7

What is the sum of the solutions to $(2x - 1)^2 = (x + 2)^2$?

If a and b are real numbers and $a^2 = b^2$, then either $a = b$ or $a = -b$. It follows then that since $(2x - 1)^2 = (x + 2)^2$, either $2x - 1 = x + 2$ or $2x - 1 = -(x + 2)$. In the first case, subtracting x and adding 1 to both sides of the equation yields $x = 3$. In the second case, applying the distributive property on the right side of the equation gives $2x - 1 = -x - 2$.

Adding x and 1 to both sides of this equation yields $3x = -1$, or $x = -\frac{1}{3}$.

Therefore, the sum of the solutions of $(2x - 1)^2 = (x + 2)^2$ is $3 + \left(-\frac{1}{3}\right) = \frac{8}{3}$.

Exponential Functions, Equations, and Expressions and Radicals

Some Advanced Math questions may ask you to build a function that models a given context. Exponential functions model situations in which a quantity is multiplied by a constant factor for each time period. An exponential function can be increasing with time, in which case it models exponential growth, or it can be decreasing with time, in which case it models exponential decay.

Advanced Math Sample Question 8

A researcher estimates that the population of a city is increasing at an annual rate of 0.6%. If the current population of the city is 80,000, which of the following expressions appropriately models the population of the city t years from now according to the researcher's estimate?

- A) $80,000(1 + 0.006)^t$
- B) $80,000(1 + 0.006^t)$
- C) $80,000 + 1.006^t$
- D) $80,000(0.006^t)$

REMINDER

A quantity that grows or decays by a fixed percent at regular intervals is said to exhibit exponential growth or decay, respectively.

Exponential growth is represented by the function $y = a(1 + r)^t$, while exponential decay is represented by the function $y = a(1 - r)^t$, where y is the new population, a is the initial population, r is the rate of growth or decay, and t is the number of time intervals that have elapsed.

According to the researcher's estimate, the population is increasing by 0.6% each year. Since 0.6% is equal to 0.006, after the first year the population is $80,000 + 0.006(80,000) = 80,000(1 + 0.006)$.

After the second year, the population is

$$80,000(1 + 0.006) + 0.006(80,000)(1 + 0.006) = 80,000(1 + 0.006)^2.$$

Similarly, after t years, the population will be $80,000(1 + 0.006)^t$ according to the researcher's estimate. This is choice A.

A well-known example of exponential decay is the decay of a radioactive isotope. One example involves iodine-131, a radioactive isotope used in some medical treatments. The half-life of iodine-131 is 8.02 days; that is, after 8.02 days, half of the iodine-131 in a sample will have decayed. Suppose a sample with a mass of A milligrams of iodine-131 decays for d days. Every 8.02 days, the quantity of iodine-131 is multiplied by $\frac{1}{2}$, or 2^{-1} . In d days, a total of $\frac{d}{8.02}$ different 8.02-day periods will have passed, and so the original quantity will have been multiplied by 2^{-1} a total of $\frac{d}{8.02}$ times. Therefore, the mass, in milligrams, of iodine-131 remaining in the sample will be $A(2^{-1})^{\frac{d}{8.02}} = A(2^{-\frac{d}{8.02}})$.

In the preceding discussion, we used the identity $\frac{1}{2} = 2^{-1}$. Advanced Math questions may require you to apply this and other laws of exponents as well as the relationship between powers and radicals.

Some Advanced Math questions may ask you to use properties of exponents to rewrite expressions.

Advanced Math Sample Question 9

Which of the following is equivalent to $\left(\frac{1}{\sqrt{x}}\right)^n$, where $x > 0$?

- A) $x^{\frac{n}{2}}$
- B) $x^{-\frac{n}{2}}$
- C) $x^{n+\frac{1}{2}}$
- D) $x^{n-\frac{1}{2}}$

The expression $\sqrt{x}$ is equal to $x^{\frac{1}{2}}$. Thus, $\frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$, and $\left(\frac{1}{\sqrt{x}}\right)^n = \left(x^{-\frac{1}{2}}\right)^n = x^{-\frac{n}{2}}$. Choice B is the correct answer.

An Advanced Math question may also ask you to solve a radical equation. In solving radical equations, you may square both sides of an equation. Since squaring both sides of an equation may result in an extraneous solution, you may end up with a root to the simplified equation that isn't a root to the original equation. Thus, when solving a radical equation, you must check any solution you get in the original equation.

Advanced Math Sample Question 10

$$x - 12 = \sqrt{x + 44}$$

What are all possible solutions to the given equation?

- A) 5
- B) 20
- C) -5 and 20
- D) 5 and 20

Squaring each side of $x - 12 = \sqrt{x + 44}$ gives

$$(x - 12)^2 = (\sqrt{x + 44})^2$$

$$(x - 12)^2 = x + 44$$

$$x^2 - 24x + 144 = x + 44$$

$$x^2 - 25x + 100 = 0$$

$$(x - 5)(x - 20) = 0$$

The solutions to this quadratic equation are $x = 5$ and $x = 20$. However, since the first step was to square each side of the given equation, which may have introduced an extraneous solution, you need to check $x = 5$ and $x = 20$ in the original equation. Substituting 5 for x gives

$$5 - 12 = \sqrt{5 + 44}$$

$$-7 = \sqrt{49}$$

This isn't a true statement (since $\sqrt{49}$ represents the principal square root, or only the positive square root, 7), so $x = 5$ is *not* a solution to $x - 12 = \sqrt{x + 44}$. Substituting 20 for x gives

$$20 - 12 = \sqrt{20 + 44}$$

$$8 = \sqrt{64}$$

This is a true statement, so $x = 20$ is a solution to $x - 12 = \sqrt{x + 44}$. Therefore, the only solution to the given equation is 20, which is choice B.

STRATEGY

Practice your exponent rules.

Know, for instance, that $\sqrt{x} = x^{\frac{1}{2}}$ and that $\frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$.

STRATEGY

A good strategy to use when solving radical equations is to square both sides of the equation. When doing so, however, be sure to check the solutions in the original equation, as you may end up with a root that isn't a solution to the original equation.

Solving Rational Equations

Questions in the Advanced Math domain may assess your ability to work with rational expressions, including fractions with a variable in the denominator. This may include finding the solution to a rational equation.

Advanced Math Sample Question 11

$$\frac{3}{t+1} = \frac{2}{t+3} + \frac{1}{4}$$

If t is a solution to the given equation and $t > 0$, what is the value of t ?

STRATEGY

When solving for a variable in an equation involving fractions, a good first step is to clear the variable out of the denominators of the fractions. Remember that you can only multiply both sides of an equation by an expression when you know the expression can't be equal to 0.

Since $t > 0$, both sides of the given equation can be multiplied by the lowest common denominator, which is $4(t+1)(t+3)$. The resulting equivalent equation won't have any fractions, and the variable will no longer be in the denominator. This gives $12(t+3) = 8(t+1) + (t+1)(t+3)$. This equation can be rewritten as $12t + 36 = (8t + 8) + (t^2 + 4t + 3)$, or $12t + 36 = t^2 + 12t + 11$. Subtracting $12t$ and 36 from both sides of this equation yields $0 = t^2 - 25$. This equation can be rewritten by factoring the right-hand side as $0 = (t-5)(t+5)$. By the zero product property, $t-5 = 0$ or $t+5 = 0$. Therefore, the solutions to the equation are $t = 5$ or $t = -5$. Since $t > 0$, the value of t is 5.

Systems of Equations

Questions in the Advanced Math domain may ask you to solve a system of equations in two variables in which one equation is linear and the other equation is quadratic or another nonlinear equation.

Advanced Math Sample Question 12

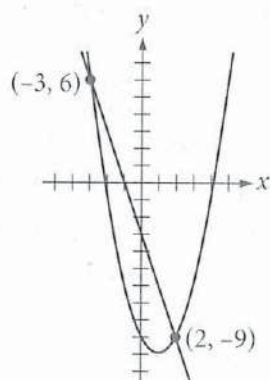
$$\begin{aligned} 3x + y &= -3 \\ (x+1)^2 - 4(x+1) - 6 &= y \end{aligned}$$

If (x, y) is a solution to the given system of equations and $y > 0$, what is the value of y ?

STRATEGY

The first step used to solve this example was substitution, an approach you may use on questions about finding the solution to a system of linear equations as well. The second step used to solve this example was taken after noticing that $(x+1)$ can be treated as a variable.

One method for solving systems of equations is substitution. If the first equation is solved for y , it can be substituted in the second equation. Subtracting $3x$ from each side of the first equation gives you $y = -3 - 3x$, which can be rewritten as $y = -3(x+1)$. Substituting $-3(x+1)$ for y in the second equation gives you $(x+1)^2 - 4(x+1) - 6 = -3(x+1)$. Since the factor $(x+1)$ appears as a squared term and a linear term, the equation can be thought of as a quadratic equation in the variable $(x+1)$, so collecting the terms and setting the expression equal to 0 gives you $(x+1)^2 - (x+1) - 6 = 0$. Rewriting this equation by factoring gives you $((x+1)-3)((x+1)+2) = 0$, or $(x-2)(x+3) = 0$. Thus, either $x = 2$, which gives $y = -3 - 3(2) = -9$; or $x = -3$, which gives $y = -3 - 3(-3) = 6$. Therefore, the solutions to the system are $(2, -9)$ and $(-3, 6)$. Since the question states that $y > 0$, the value of y is 6.



The solutions of the system are given by the intersection points of the two graphs. Questions in the digital SAT Math section may assess this or other relationships between algebraic and graphical representations of functions.

Relationships between Algebraic and Graphical Representations of Functions

A function f has a graph in the xy -plane, which is the graph of the equation $y = f(x)$, or, equivalently, consists of all ordered pairs $(x, f(x))$. Some Advanced Math questions may assess your ability to relate properties of the function f to properties of its graph and vice versa. You may be required to apply some of the following relationships:

- **Intercepts.** The x -intercepts of the graph of f correspond to values of x such that $f(x) = 0$, which corresponds to values of x where the graph intersects with the x -axis, also known as the function's zeros; if the function f has no zeros, its graph has no x -intercepts and vice versa. The y -intercept of the graph of f corresponds to the value of $f(0)$, or the value of y where the graph intersects with the y -axis. If $x = 0$ is not in the domain of f , the graph of f has no y -intercept and vice versa.
- **Domain and range.** The domain of f is the set of all x for which $f(x)$ is defined. The range of f is the set of all y such that $y = f(x)$ for some value of x in the domain. The domain and range can be found from the graph of f as the set of all x -coordinates and y -coordinates, respectively, of points on the graph.
- **Maximum and minimum values.** The maximum and minimum values of f can be found by locating the highest and the lowest points on the graph, respectively. For example, suppose P is the highest point on the graph of f . Then the y -coordinate of P is the maximum value of f , and the x -coordinate of P is where f takes on its maximum value.
- **Increasing and decreasing.** The graph of f shows the intervals over which the function f is increasing and decreasing.

REMINDER

The domain of a function is the set of all values for which the function is defined. The range of a function is the set of all values that correspond to the values in the domain, given the relationship defined by the function, or the set of all outputs that are associated with all the possible inputs.

STRATEGY

Don't assume the size of the units on the two axes are equal unless the question states they're equal or you can conclude they're equal from the information given.

- **End behavior.** The graph of f can indicate if $f(x)$ increases or decreases without limit as x increases or decreases without limit.
- **Transformations.** For a graph of a function f , a change of the form $f(x) + a$ will result in a vertical shift of a units, and a change of the form $f(x - a)$ will result in a horizontal shift of a units.

Note: The Math section uses the following conventions about graphs in the xy -plane unless a particular question clearly states or shows a different convention:

- The axes are perpendicular.
- Scales on the axes are linear scales.
- The size of the units on the two axes can't be assumed to be equal unless the question states they're equal or you're given enough information to conclude they're equal.
- The values on the horizontal axis increase as you move to the right.
- The values on the vertical axis increase as you move up.

Advanced Math Sample Question 13

The graph of which of the following functions in the xy -plane has x -intercepts at -4 and 5 ?

- A) $f(x) = (x + 4)(x - 5)$
- B) $g(x) = (x - 4)(x + 5)$
- C) $h(x) = (x - 4)^2 + 5$
- D) $k(x) = (x + 5)^2 - 4$

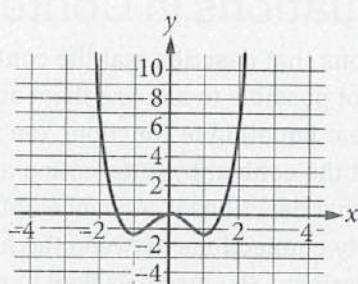
The x -intercepts of the graph of a function correspond to the zeros of the function. All the functions in the choices are defined by quadratic equations, so the answer must be a quadratic function. If a quadratic function has x -intercepts at -4 and 5 , then the values of the function at -4 and 5 are each 0 ; that is, the zeros of the function occur at $x = -4$ and at $x = 5$. Since the function is defined by a quadratic equation and has zeros at $x = -4$ and $x = 5$, it must have $(x + 4)$ and $(x - 5)$ as factors. Therefore, choice A, $f(x) = (x + 4)(x - 5)$, is correct.

The graph in the xy -plane of each of the functions in the previous example is a parabola. Using the defining equations, you can tell that the graph of g has x -intercepts at 4 and -5 ; the graph of h has its vertex at $(4, 5)$; and the graph of k has its vertex at $(-5, -4)$.

STRATEGY

Another way to think of sample 13 is to ask yourself, "Which answer choice represents a function that has values of 0 when $x = -4$ and $x = 5$?"

Advanced Math Sample Question 14



The function $f(x) = x^4 - 2.4x^2$ is graphed in the xy -plane where $y = f(x)$, as shown. If k is a constant such that the equation $f(x) = k$ has 4 solutions, which of the following could be the value of k ?

- A) 1
- B) 0
- C) -1
- D) -2

Choice C is correct. Since $f(x) = x^4 - 2.4x^2$, the equation $f(x) = k$, or $x^4 - 2.4x^2 = k$, will have four solutions if and only if the graph of the horizontal line with equation $y = k$ intersects the graph of f at four points. The graph shows that of the given choices, only for choice C, -1, does the graph of $y = -1$ intersect the graph of f at four points.

Function Notation

The Math section may assess your understanding of function notation. In such cases, you must be able to evaluate a function given the rule that defines it, and if the function describes a context, you may need to interpret the value of the function in the context. A question may ask you to interpret a function when an expression, such as $2x$ or $x + 1$, is used as the argument instead of the variable x .

Advanced Math Sample Question 15

If $g(x) = 2x + 1$ and $f(x) = g(x) + 4$, what is $f(2)$?

You're given $f(x) = g(x) + 4$; therefore, $f(2) = g(2) + 4$. To determine the value of $g(2)$, use the function $g(x) = 2x + 1$. Thus, $g(2) = 2(2) + 1$, and therefore, $g(2) = 5$. Substituting $g(2)$ gives $f(2) = 5 + 4$, or $f(2) = 9$.

Alternatively, since $f(x) = g(x) + 4$ and $g(x) = 2x + 1$, it follows that $f(x)$ must equal $2x + 1 + 4$, or $2x + 5$. Therefore, $f(2) = 2(2) + 5 = 9$.

STRATEGY

What may seem at first to be a complex question could boil down to straightforward substitution.

Interpreting and Analyzing More Complex Equations in Context

Equations and functions that describe real-life contexts can be complex. Often, it's not possible to analyze them as completely as you can analyze a linear equation or function. You still can acquire key information about the context by interpreting and analyzing the equation or function that describes it. Advanced Math questions may ask you to identify connections between the function, its graph, and the context it describes. You may be asked to use an equation describing a context to determine how a change in one quantity affects another quantity. You may also be asked to manipulate an equation to isolate a quantity of interest on one side of the equation. You may be asked to produce or identify a form of an equation that reveals new information about the context it represents or about the graphical representation of the equation.

Advanced Math Sample Question 16

For a certain reservoir, the function f gives the water level $f(n)$, to the nearest whole percent of capacity, on the n th day of 2016. Which of the following is the best interpretation of $f(37) = 70$?

- A) The water level of the reservoir was at 37% capacity for 70 days in 2016.
- B) The water level of the reservoir was at 70% capacity for 37 days in 2016.
- C) On the 37th day of 2016, the water level of the reservoir was at 70% capacity.
- D) On the 70th day of 2016, the water level of the reservoir was at 37% capacity.

The function f gives the water level, to the nearest whole percent of capacity, on the n th day of 2016. It follows that $f(37) = 70$ means that on the 37th day of 2016, the water level of the reservoir was at 70% capacity. This statement is choice C.

Advanced Math Sample Question 17

If an object of mass m is moving at speed v , the object's kinetic energy (KE) is given by the equation $KE = \frac{1}{2}mv^2$. If the mass of the object is halved and its speed is doubled, how does the kinetic energy change?

- A) The kinetic energy is halved.
- B) The kinetic energy is unchanged.
- C) The kinetic energy is doubled.
- D) The kinetic energy is quadrupled (multiplied by a factor of 4).

Choice C is correct. If the mass of the object is halved, the new mass is $\frac{m}{2}$. If the speed of the object is doubled, its new speed is $2v$. Therefore, the new kinetic energy is $\frac{1}{2} \left(\frac{m}{2} \right) (2v)^2 = \frac{1}{2} \left(\frac{m}{2} \right) (4v^2) = mv^2$. This is double the original kinetic energy of the object, which was $\frac{1}{2}mv^2$.

STRATEGY

Another way to check your answer in sample 17 is to substitute simple numerical values for the variables m , v , and KE when those values are altered as indicated by the question. If the value 1 is substituted for both m and v , then KE is $\frac{1}{2}$. However, substituting $\frac{1}{2}$ for m and 2 for v yields a value for KE of 1. Since 1 is twice the value of $\frac{1}{2}$, you know that KE is doubled.

Advanced Math Sample Question 18

A gas in a container will escape through holes of microscopic size, as long as the holes are larger than the gas molecules. This process is called effusion. If a gas of molar mass M_1 effuses at a rate of r_1 and a gas of molar mass M_2 effuses at a rate of r_2 , then the following relationship holds.

$$\frac{r_1}{r_2} = \sqrt{\frac{M_2}{M_1}}$$

This is known as Graham's law. Which of the following correctly expresses M_2 in terms of M_1 , r_1 , and r_2 ?

- A) $M_2 = M_1 \left(\frac{r_1^2}{r_2^2} \right)$
- B) $M_2 = M_1 \left(\frac{r_2^2}{r_1^2} \right)$
- C) $M_2 = \sqrt{M_1} \left(\frac{r_1}{r_2} \right)$
- D) $M_2 = \sqrt{M_1} \left(\frac{r_2}{r_1} \right)$

Squaring each side of $\frac{r_1}{r_2} = \sqrt{\frac{M_2}{M_1}}$ gives $\left(\frac{r_1}{r_2} \right)^2 = \left(\sqrt{\frac{M_2}{M_1}} \right)^2$, which can be rewritten as $\frac{M_2}{M_1} = \frac{r_1^2}{r_2^2}$. Multiplying each side of $\frac{M_2}{M_1} = \frac{r_1^2}{r_2^2}$ by M_1 gives $M_2 = M_1 \left(\frac{r_1^2}{r_2^2} \right)$, which is choice A.

Advanced Math Sample Question 19

A store manager estimates that if a video game is sold at a price of p dollars, the store will have weekly revenue, in dollars, of $r(p) = -4p^2 + 200p$ from the sale of the video game. Which of the following equivalent forms of $r(p)$ shows, as constants or coefficients, the maximum possible weekly revenue and the price that results in the maximum revenue?

- A) $r(p) = 200p - 4p^2$
- B) $r(p) = -2(2p^2 - 100p)$
- C) $r(p) = -4(p^2 - 50p)$
- D) $r(p) = -4(p - 25)^2 + 2,500$

Choice D is correct. The graph of r in the coordinate plane is a parabola that opens downward. The maximum value of revenue corresponds to the vertex of the parabola. Since the square of any real number is always nonnegative, the form $r(p) = -4(p - 25)^2 + 2,500$ shows that the vertex of the parabola is $(25, 2,500)$; that is, the maximum must occur where $-4(p - 25)^2$ is 0, which is $p = 25$, and this maximum is $r(25) = 2,500$. Thus, the maximum possible weekly revenue and the price that results in the maximum revenue occur as constants in the form $r(p) = -4(p - 25)^2 + 2,500$.

STRATEGY

Always start by identifying exactly what the question asks. In sample 18, you're being asked to isolate the variable M_2 . Squaring both sides of the equation is a great first step, as it allows you to eliminate the radical sign.

STRATEGY

The fact that the coefficient of the squared term is negative for this function indicates that the graph of r in the coordinate plane is a parabola that opens downward. Thus, the maximum value of revenue corresponds to the vertex of the parabola.

Summary: Advanced Math

Main purpose: To assess your understanding of nonlinear relationships

Proportion of the test section: About 35 percent (approximately thirteen to fifteen questions)

Question types

- Equivalent expressions
- Nonlinear equations in one variable and systems of equations in two variables
- Nonlinear functions



CHAPTER 24

Math: Questions— Advanced Math Drills

1

Which of the following expressions is equivalent to $2(ab - 3) + 2$?

- A) $2ab - 1$
- B) $2ab - 4$
- C) $2ab - 5$
- D) $2ab - 8$

2

$$f(x) = (x + 0.25x)(50 - x)$$

The function f is defined as shown. What is the value of $f(20)$?

- A) 250
- B) 500
- C) 750
- D) 2,000

3

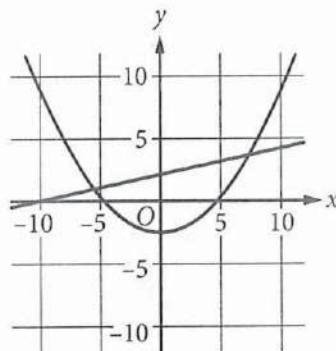
$$x^2 + 6x + 9 = 36$$

What is the positive solution to the given equation?

4

The function f is defined by $f(x) = \frac{22}{x+1}$. What is the value of $f(22)$?

5



A system of equations consists of a quadratic equation and a linear equation. The equations in this system are graphed in the xy -plane shown. How many solutions does this system have?

- A) 0
- B) 1
- C) 2
- D) 3

6

$$S = 4\pi r^2$$

The formula shown gives the surface area, S , of a sphere in terms of the length of its radius, r . Which of the following gives the radius of the sphere in terms of its surface area?

- A) $r = \sqrt{\frac{S}{4\pi}}$
- B) $r = \sqrt{\frac{4\pi}{S}}$
- C) $r = \frac{\sqrt{S}}{4\pi}$
- D) $r = \frac{\sqrt{4\pi}}{S}$

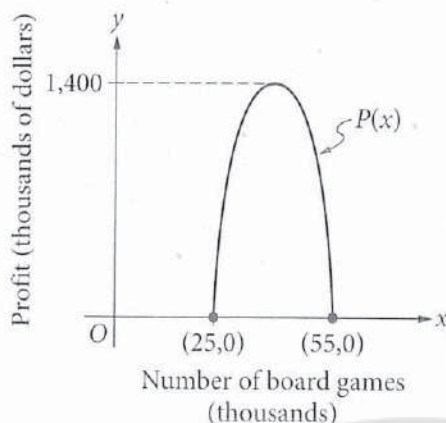
7

$$\frac{4x}{2(x^2 - 1)} - \frac{3x}{3(x^2 - 1)}$$

Which of the following is equivalent to the given expression for $x \neq -1$ and $x \neq 1$?

- A) $\frac{1}{6(x - 1)}$
- B) $\frac{x}{6(x^2 - 1)}$
- C) $\frac{1}{x - 1}$
- D) $\frac{x}{x^2 - 1}$

8



A company produces board games and sells them online and in stores. The quadratic function P models the company's monthly profits $P(x)$, in thousands of dollars, when x board games, in thousands, are produced and sold. The graph of $y = P(x)$, where $25 \leq x \leq 55$, is shown in the xy -plane. How many board games must the company produce and sell in order to earn the maximum profit estimated by the model?

- A) 20,000
- B) 40,000
- C) 60,000
- D) 1,400,000

9

$$\begin{aligned} y - x &= 30 \\ y &= x^2 - 28x \end{aligned}$$

The graphs of the equations in the given system intersect at the point (x, y) in the xy -plane. What is a possible value of y ?

10

In the equation $9x^2 + 108x + \frac{c}{4} = 0$, c is a constant. If the equation has exactly one real solution, what is the value of c ?

- A) 0
- B) 324
- C) 1,296
- D) 11,664

11

Kao measured the temperature of a cup of hot chocolate placed in a room with a constant temperature of 70 degrees Fahrenheit ($^{\circ}\text{F}$). The temperature of the hot chocolate was 185°F at 6:00 p.m. when it started cooling. The temperature of the hot chocolate was 156°F at 6:05 p.m. and 135°F at 6:10 p.m. The hot chocolate's temperature continued to decrease. Of the following functions, which best models the temperature $T(m)$, in degrees Fahrenheit, of Kao's hot chocolate m minutes after it started cooling?

- A) $T(m) = 185(1.25)^m$
- B) $T(m) = 185(0.85)^m$
- C) $T(m) = (185 - 70)(0.75)^{\frac{m}{5}}$
- D) $T(m) = 70 + 115(0.75)^{\frac{m}{5}}$

12

$$\sqrt{5(x - k)} = x - k$$

In the given equation, k is a positive constant. The greatest solution to the equation is 12. What is the value of k ?

13

$$g(x) = (5 - 2x)(14 + 2x)$$

The function g is defined by the given equation. For what value of x does $g(x)$ reach its maximum?

14

A rectangular volleyball court has an area of 162 square meters. If the length of the court is twice the width, what is the width of the court, in meters?

- A) 9
- B) 18
- C) 27
- D) 54

15

$$x^2 = 6x + y$$

$$y = -6x + 36$$

A solution to the given system of equations is (x, y) . Which of the following is a possible value of xy ?

- A) 0
- B) 6
- C) 12
- D) 36

16

The function f is defined by $f(x) = ax^2 + bx + c$, where a , b , and c are constants and $1 < a < 4$. The graph of $y = f(x)$ in the xy -plane passes through points $(11, 0)$ and $(-2, 0)$. If a is an integer, what could be the value of $a + b$?



Answer Explanations

QUESTION 1

Choice B is correct. Applying the distributive property to the given expression yields $2(ab) + 2(-3) + 2$, or $2ab - 6 + 2$. Adding the like terms -6 and 2 results in the expression $2ab - 4$.

Choice A is incorrect. This expression is equivalent to $2(ab - 3) + 5$. **Choice C** is incorrect. This expression is equivalent to $2(ab - 3) + 1$. **Choice D** is incorrect. This expression is equivalent to $2(ab - 3) - 2$.

QUESTION 2

Choice C is correct. Adding the like terms x and $0.25x$ yields the equation $f(x) = (1.25x)(50 - x)$. Substituting 20 for x yields $f(x) = (1.25(20))(50 - 20)$ or $f(20) = (25)(30)$, which is equivalent to $f(20) = 750$.

Choices A, B, and D are incorrect and may result from conceptual or calculation errors.

QUESTION 3

The correct answer is 3 . Subtracting 36 from both sides of the given equation yields $x^2 + 6x - 27 = 0$. Two numbers whose product is -27 and whose sum is 6 are 9 and -3 . Therefore, by factoring, the equation $x^2 + 6x - 27 = 0$ can be rewritten as $(x + 9)(x - 3) = 0$. By the zero product property, it follows that $x + 9 = 0$ or $x - 3 = 0$. Subtracting 9 from both sides of the equation $x + 9 = 0$ yields $x = -9$. Adding 3 to both sides of the equation $x - 3 = 0$ yields $x = 3$. Therefore, the two solutions to the given equation are -9 and 3 . Thus, the positive solution to the given equation is 3 .

Alternate approach: The left-hand side of the given equation is a perfect square trinomial. It follows that the given equation can be rewritten as $(x + 3)^2 = 36$. Taking the square root of both sides of this equation yields two equations: $x + 3 = 6$ and $x + 3 = -6$. Subtracting 3 from both sides of the equation $x + 3 = 6$ yields $x = 3$. Subtracting 3 from both sides of the equation $x + 3 = -6$ yields $x = -9$. Therefore, the two solutions to the given equation are 3 and -9 . Thus, the positive solution to the given equation is 3 .

QUESTION 4

The correct answer is $\frac{22}{23}$. For the given function f , the value of $f(22)$ is the value of $f(x)$ when $x = 22$. Substituting 22 for x in the equation $f(x) = \frac{22}{x+1}$ yields $f(22) = \frac{22}{(22)+1}$, or $f(22) = \frac{22}{23}$. Therefore, the value of $f(22)$ is $\frac{22}{23}$. Note that $\frac{22}{23}$, $.9565$, 0.956 , and 0.957 are examples of ways to enter a correct answer.

QUESTION 5

Choice C is correct. The solutions to a system of two equations correspond to points where the graphs of the equations intersect. The given graphs intersect at 2 points; therefore, the system has 2 solutions.

Choices A and B are incorrect because the given graphs intersect at more than one point. **Choice D** is incorrect. It's not possible for the graph of a quadratic equation and the graph of a linear equation to intersect at more than two points.

QUESTION 6

Choice A is correct. Solving the given formula for r yields the radius of the sphere in terms of its surface area. Dividing both sides of the given equation by 4π yields $\frac{S}{4\pi} = r^2$. Taking the square root of both sides of this equation yields

$$\sqrt{\frac{S}{4\pi}} = r, \text{ which can be rewritten as } r = \sqrt{\frac{S}{4\pi}}.$$

Choices *B*, *C*, and *D* are incorrect and may result from conceptual or calculation errors.

QUESTION 7

Choice D is correct. Multiplying the first fraction in the given expression by $\frac{3}{3}$ and multiplying the second fraction in the given expression by $\frac{2}{2}$

results in both fractions having a common denominator:

$$\frac{4x}{2(x^2 - 1)} - \frac{3x}{3(x^2 - 1)} = \frac{(3)(4x)}{(3)(2)(x^2 - 1)} - \frac{2(3x)}{(2)(3)(x^2 - 1)}. \text{ Rewriting the right-hand side of this equation yields } \frac{12x}{6(x^2 - 1)} - \frac{6x}{6(x^2 - 1)}, \text{ which is equivalent to } \frac{12x - 6x}{6(x^2 - 1)} \text{ or } \frac{6x}{6(x^2 - 1)}.$$

Rewriting this fraction yields $\frac{x}{x^2 - 1}$.

Alternate approach: The given expression can be rewritten as $\frac{2x}{x^2 - 1} - \frac{x}{x^2 - 1}$.

Since the fractions in this expression have a common denominator of $x^2 - 1$,

it follows that this expression is equivalent to $\frac{2x - x}{x^2 - 1}$, or $\frac{x}{x^2 - 1}$.

Choices *A*, *B*, and *C* are incorrect and may result from conceptual or calculation errors.

QUESTION 8

Choice B is correct. For each point (x, y) on the graph shown, the x -coordinate gives the number of board games, in thousands, that the company produces and sells, and the y -coordinate gives the profit, in thousands of dollars, from the sale of these board games. The maximum profit is represented by the point on the graph with the greatest y -coordinate. Because P is a quadratic function, the graph of $y = P(x)$ is a parabola and the point with the greatest y -coordinate is the vertex. Because of the symmetry of the parabola, the x -coordinate of its vertex is the mean of the x -coordinates of any two symmetrical points on the graph. For example, the x -intercepts $(25, 0)$ and $(55, 0)$ are symmetrical points on the graph. Therefore, the x -coordinate of the vertex is $\frac{25 + 55}{2}$, or 40. It follows that the vertex of the graph is $(40, 1,400)$. This means that the maximum profit of \$1,400,000 is reached when the company produces and sells 40,000 board games.

Choices *A* and *C* are incorrect and may result from conceptual or calculation errors. *Choice D* is incorrect and may result from finding the maximum profit, in dollars, rather than the number of board games that need to be produced and sold to reach the maximum profit.

QUESTION 9

The correct answer is either 29 or 60. It's given that the graphs of the equations in the given system intersect at the point (x, y) in the xy -plane. Therefore, this intersection point represents a solution to the given system. The first equation, $y - x = 30$, can be rewritten as $y = x + 30$. Substituting $x + 30$ for y in the second equation, $y = x^2 - 28x$, yields $x + 30 = x^2 - 28x$. Subtracting x and 30 from both sides of this equation yields $0 = x^2 - 28x - x - 30$, or $0 = x^2 - 29x - 30$, which is equivalent to $0 = (x + 1)(x - 30)$. It follows that $x + 1 = 0$ or $x - 30 = 0$. Therefore, $x = -1$ or $x = 30$. Substituting -1 for x in the equation $y - x = 30$ yields

$y - (-1) = 30$, or $y + 1 = 30$, which is equivalent to $y = 29$. Therefore, the graphs of the equations in the given system intersect at the point $(-1, 29)$. Substituting 30 for x in the equation $y - x = 30$ yields $y - 30 = 30$, which is equivalent to $y = 60$. Therefore, the graphs of the equations in the given system intersect at the point $(30, 60)$. Thus, either 29 or 60 is a possible value of y . Note that 29 and 60 are examples of ways to enter a correct answer.

QUESTION 10

Choice C is correct. A quadratic equation of the form $Ax^2 + Bx + C = 0$, where A , B , and C are constants, has exactly one solution if and only if its discriminant, $B^2 - 4AC$, is equal to zero. The given equation is in this form, where $A = 9$,

$B = 108$, and $C = \frac{c}{4}$. Substituting 9 for A , 108 for B , and $\frac{c}{4}$ for C into $B^2 - 4AC$ yields $(108)^2 - 4(9)(\frac{c}{4})$, or $11,664 - 9c$. Therefore, if the given equation has

exactly one solution, then $11,664 - 9c = 0$. Adding $9c$ to both sides of this equation yields $11,664 = 9c$. Dividing both sides of this equation by 9 yields $1,296 = c$. Therefore, if the equation has exactly one real solution, the value of c is 1,296.

Choice A is incorrect. This is the value of $9x^2 + 108x + \frac{c}{4}$, not c . **Choice B** is incorrect. This is the value of $\frac{c}{4}$, not c . **Choice D** is incorrect. This is the value of $(108)^2$, not c .

QUESTION 11

Choice D is correct. The temperature of the cup of hot chocolate, $T(m)$, will decrease as the number of minutes, m , after it starts cooling increases, since the temperature of the room is lower than the starting temperature of the cup of hot chocolate. Each of the answer choices involves the variable m in an exponent. Choices B, C, and D each have a function with the base of the exponent less than 1, so each of them is a decreasing function, where the value of $T(m)$ will decrease over time. It's given that the temperature of the hot chocolate was 185°F at 6:00 p.m. when it started cooling. It follows that $T(0) = 185$. It's also given that at 6:05 p.m., or 5 minutes after it started cooling, the temperature of the hot chocolate was 156°F and that at 6:10 p.m., or 10 minutes after it started cooling, the temperature was 135°F . Therefore, $T(5) = 156$ and $T(10) = 135$. For the function in choice D, substituting 0, 5, and 10 for m in the function yields $T(0) = 70 + 115(0.75)^{\frac{0}{5}}$, or 185; $T(5) = 70 + 115(0.75)^{\frac{5}{5}}$, which is approximately 156; and $T(10) = 70 + 115(0.75)^{\frac{10}{5}}$, which is approximately 135. Therefore, of the given choices, choice D best models the temperature of Kao's hot chocolate m minutes after it started cooling.

Choice A is incorrect because it's a function with the base of the exponent equal to 1.25, which results in the value of $T(m)$ increasing over time, rather than decreasing. **Choice B** is incorrect because substituting 0, 5, and 10 for m in this function yields approximately 185, 82, and 36, respectively, rather than 185, 156, and 135. **Choice C** is incorrect because substituting 0, 5, and 10 for m in this function yields approximately 115, 86, and 65, respectively, rather than 185, 156, and 135.

QUESTION 12

The correct answer is 7. Squaring both sides of the given equation yields $5(x - k) = (x - k)^2$, or $5(x - k) = (x - k)(x - k)$. Subtracting $5(x - k)$ from both sides of this equation yields $0 = (x - k)(x - k) - 5(x - k)$. Factoring out the common factor of $x - k$ from the terms on the right-hand side of this equation yields $0 = (x - k)(x - k - 5)$, or $0 = (x - k)(x - (k + 5))$. By the zero product property,

it follows that $x - k = 0$ or $x - (k + 5) = 0$. Adding k to both sides of the equation $x - k = 0$ yields $x = k$. Adding $(k + 5)$ to both sides of the equation $x - (k + 5) = 0$ yields $x = k + 5$. Thus, the two solutions to the given equation are k and $k + 5$. It follows that the greatest solution to the given equation is $k + 5$. It's given that the greatest solution to the given equation is 12. Therefore, $k + 5 = 12$. Subtracting 5 from both sides of this equation yields $k = 7$. Thus, the value of k is 7.

QUESTION 13

The correct answer is $-\frac{9}{4}$. It's given that $g(x) = (5 - 2x)(14 + 2x)$, which can be rewritten as $g(x) = -4x^2 - 18x + 70$. Since the coefficient of the x^2 -term is negative, the graph of $y = g(x)$ in the xy -plane opens downward and reaches a maximum value at its vertex. The x -coordinate of the vertex is the value of x such that $g(x)$ reaches its maximum. For the graph of an equation in the form $g(x) = ax^2 + bx + c$, where a , b , and c are constants, the x -coordinate of the vertex is $-\frac{b}{2a}$. For the equation $g(x) = -4x^2 - 18x + 70$, $a = -4$, $b = -18$, and $c = 70$. It follows that the x -coordinate of the vertex is $-\frac{-18}{2(-4)}$, or $-\frac{9}{4}$. Therefore, $g(x)$ reaches its maximum when the value of x is $-\frac{9}{4}$. Note that $-9/4$ and -2.25 are examples of ways to enter a correct answer.

QUESTION 14

Choice A is correct. It's given that the volleyball court is rectangular and has an area of 162 square meters. The formula for the area of a rectangle is $A = l \cdot w$, where A is the area, l is the length, and w is the width of the rectangle. It's also given that the length of the volleyball court is twice the width, thus $l = 2w$. Substituting 162 for A in the formula for the area of a rectangle and using the relationship between length and width for this rectangle yields $162 = (2w)(w)$. This equation can be rewritten as $162 = 2w^2$. Dividing both sides of this equation by 2 yields $81 = w^2$. Taking the square root of both sides of the equation yields $\pm 9 = w$. Since the width of the rectangle is a positive number, the width of the volleyball court is 9 meters.

Choice B is incorrect. This is the length, not the width, of the court. **Choice C** is incorrect and may result from using 162 as the perimeter, not the area, of the court. **Choice D** is incorrect and may result from solving for w in the equation $162 = 2w + w$ instead of $162 = (2w)(w)$.

QUESTION 15

Choice A is correct. Solutions to the given systems of equations are ordered pairs (x, y) that satisfy both equations in the system. Adding the left-hand and right-hand sides of the equations in the given system yields $x^2 + y = 6x + -6x + y + 36$, or $x^2 + y = y + 36$. Subtracting y from both sides of this equation yields $x^2 = 36$. Taking the square root of both sides of this equation yields $x = 6$ and $x = -6$. Therefore, there are two solutions to this system of equations, one with an x -coordinate of 6 and the other with an x -coordinate of -6 . Substituting 6 for x in the second equation yields $y = -6(6) + 36$, or $y = 0$; therefore, one solution is $(6, 0)$. Similarly, substituting -6 for x in the second equation yields $y = -6(-6) + 36$, or $y = 72$; therefore, the other solution is $(-6, 72)$. Of these possible values of xy , only 0 is given as a choice.

Choice B is incorrect. This is the x -coordinate of one of the solutions, $(6, 0)$. **Choice C** is incorrect and may result from conceptual or calculation errors. **Choice D** is incorrect. This is the square of the x -coordinate of one of the solutions, $(6, 0)$.

QUESTION 16

The correct answer is either -16 or -24 . It's given that the function f is defined by the equation $f(x) = ax^2 + bx + c$, where a , b , and c are constants, and $1 < a < 4$. It's also given that the graph of $y = f(x)$ in the xy -plane passes through the points $(11, 0)$ and $(-2, 0)$. Therefore, $f(11) = 0$ and $f(-2) = 0$. It follows that $x - 11$ and $x + 2$ are factors of $f(x)$, and the equation $f(x) = ax^2 + bx + c$ can be rewritten as $f(x) = a(x - 11)(x + 2)$. Applying the distributive property on the right-hand side of this equation yields $f(x) = a(x^2 - 9x - 22)$, or $f(x) = ax^2 - 9ax - 22a$. Since $1 < a < 4$, if a is an integer, it follows that the value of a is either 2 or 3. If $a = 2$, the equation defining f is $f(x) = (2)x^2 - 9(2)x - 22(2)$, or $f(x) = 2x^2 - 18x - 44$. Since it's given that $f(x) = ax^2 + bx + c$, it follows that $ax^2 + bx + c = 2x^2 - 18x - 44$. Therefore, if $a = 2$, then $b = -18$ and the value of $a + b$ is $2 + (-18)$, or -16 . If $a = 3$, the equation defining f is $f(x) = (3)x^2 - 9(3)x - 22(3)$, or $f(x) = 3x^2 - 27x - 66$. Since it's given that $f(x) = ax^2 + bx + c$, it follows that $ax^2 + bx + c = 3x^2 - 27x - 66$. Therefore, if $a = 3$, then $b = -27$ and the value of $a + b$ is $3 + (-27)$, or -24 . Thus, if a is an integer, then the value of $a + b$ is either -16 or -24 . Note that -16 and -24 are examples of ways to enter a correct answer.



CHAPTER 25

Math: Questions— Problem-Solving and Data Analysis

Problem-Solving and Data Analysis questions in the Math section assess your ability to use your math skills and knowledge to solve problems, many of which are set in context. Some of the questions in this content domain may ask you to create a representation of a problem, consider the units involved, pay attention to the meaning of quantities, know and use different properties of mathematical operations and representations, and apply key principles of statistics and probability. Special focus in this domain is given to mathematical models. Models are representations of real-life contexts. They help us to explain or interpret the behavior of certain components of a system and to predict results that are as yet unobserved or unmeasured. You may be asked to create and use a model and to understand the distinction between the predictions of a model and the data that have been collected.

Some of the questions in this content domain may involve quantitative reasoning about ratios, rates, percentages, and proportional relationships and may require understanding and applying unit rates. Some problems may be set in contexts, which may address topics in science, social science, and real-world settings.

Some questions may present information about the relationship between two variables in a graph, scatterplot, table, or another form and ask you to analyze and draw conclusions about the given information. These questions assess your understanding of the key properties of, and the differences between, linear, quadratic, and exponential relationships and how these properties apply to the corresponding contexts.

The Problem-Solving and Data Analysis domain may also include questions that assess your understanding of essential concepts in statistics and probability. You may be asked to analyze univariate data (data involving one variable) presented in dot plots, histograms, box plots, bar graphs, and frequency tables, or bivariate data (data involving two variables) presented in scatterplots, line graphs, and two-way tables. This includes computing, comparing, and interpreting measures of center; interpreting measures of spread; describing overall patterns; and recognizing the effects of outliers on measures

of center and spread. These questions may test your understanding of the conceptual meaning of standard deviation (although you won't be asked to calculate a standard deviation).

Other questions in this domain may ask you to estimate the probability of an event, employing different approaches, rules, or probability models. Special attention is given to the notion of conditional probability, which is tested using two-way tables and in other ways.

Some questions may present you with a description of a study and ask you to decide what conclusion is most appropriate based on the design of the study. Some questions may ask about using data from a sample to draw conclusions about an entire population. These questions may also assess conceptual understanding of the margin of error (although you won't be asked to calculate a margin of error) when a population mean or proportion is estimated from sample data. Other questions may ask about making conclusions about cause-and-effect relationships between two variables.

Problem-Solving and Data Analysis questions include both multiple-choice questions and student-produced response questions.

Let's explore the content, skills, and knowledge assessed by Problem-Solving and Data Analysis questions in the Math section.

Ratio, Proportion, Units, and Percentages

Ratio and proportion is one of the major ideas in math. Introduced well before high school, ratio and proportion is a theme throughout mathematics and has many applications in daily life, career work, and higher-level math courses.

Problem-Solving and Data Analysis Sample Question 1

On Thursday, 240 adults and children attended a show. The ratio of adults to children was 5 to 1. How many children attended the show?

- A) 40
- B) 48
- C) 192
- D) 200

REMINDER

A ratio represents a relationship between quantities, not the actual quantities themselves. Fractions are an especially effective way to represent and work with ratios.

Because the ratio of adults to children was 5 to 1, there were 5 adults for every 1 child. Thus, of every 6 people who attended the show, 5 were adults and 1 was a child. In fractions, $\frac{5}{6}$ of the 240 who attended were adults and $\frac{1}{6}$ were children. Therefore, $\frac{1}{6} \times 240$, or 40 children attended the show, which is choice A.

Ratios in the Math section may be expressed in the forms 3 to 1, 3:1, $\frac{3}{1}$, or simply 3.

Problem-Solving and Data Analysis Sample Question 2

On an architect's drawing of the floor plan for a house, 1 inch represents 3 feet. If a room is represented on the floor plan by a rectangle that has sides of lengths 3.5 inches and 5 inches, what is the actual floor area of the room, in square feet?

- A) 17.5
- B) 51.0
- C) 52.5
- D) 157.5

Because 1 inch represents 3 feet, the actual dimensions of the room are 3×3.5 , or 10.5 feet, and 3×5 , or 15 feet. Therefore, the floor area of the room is 10.5×15 , or 157.5 square feet, which is choice D.

Another classic example of a ratio is the comparison of the length of an object to the length of its shadow. At a given location and time of day, it might be true that a fence post that has a height of 4 feet casts a shadow that is 6 feet long. This ratio of the height of the object to the length of its shadow, 4 to 6, or 2 to 3, remains the same for any object at the same location and time. This could be considered a unit rate: the ratio of the height of the object to the length of its shadow would be equivalent to 1 to $\frac{2}{3}$ or the unit rate $\frac{2}{3}$ -foot change in height of the object for every 1-foot change in length of shadow. So, for example, a tree that casts a shadow that's 18 feet long has a height of $18 \times \frac{2}{3}$, or 12 feet. In this situation, in which one variable quantity is always a fixed constant times another variable quantity, the two quantities are said to be directly proportional.

Thus, if variables x and y are said to be directly proportional, then there exists some number k such that $y = kx$, where k is a nonzero constant. The constant k is called the constant of proportionality.

In the preceding example, you'd say that the height of an object is directly proportional to the length of the object's shadow, with constant of proportionality $\frac{2}{3}$. So if you let L be the length of the shadow and H be the height of the object, then $H = \frac{2}{3}L$.

Notice that both L and H are lengths, so the constant of proportionality, $\frac{H}{L} = \frac{2}{3}$, has no units. In contrast, let's consider sample 2 again. On the scale drawing, 1 inch represents 3 feet. The length of an actual measurement is directly proportional to its length on the scale drawing. But to find the constant of proportionality, you need to keep track of units: $\frac{3 \text{ feet}}{1 \text{ inch}} = \frac{36 \text{ inches}}{1 \text{ inch}} = 36$. Hence, if S is a length on the scale drawing that corresponds to an actual length of R , then $R = 36S$, where R and S have the same units.

Many questions in the Math section require you to pay attention to units. Some questions in the Problem-Solving and Data Analysis content domain require you to convert units either between the U.S. customary system and the metric system or within those systems.

Problem-Solving and Data Analysis Sample Question 3

Scientists estimate that the Pacific Plate, one of Earth's tectonic plates, has moved about 1,060 kilometers in the past 10.3 million years. About how far, in miles, has the Pacific Plate moved during this same time period? (Use 1 mile = 1.6 kilometers.)

- A) 165
- B) 398
- C) 663
- D) 1,696

STRATEGY

Pay close attention to units, and convert units if required by the question. Writing out the unit conversion as a series of multiplication steps, as seen here, will help ensure accuracy. Intermediate units should cancel (as do the kilometers in sample 3), leaving you with the desired unit.

Because 1 mile = 1.6 kilometers, the distance is

$$1,060 \text{ kilometers} \times \frac{1 \text{ mile}}{1.6 \text{ kilometers}} = 662.5 \text{ miles, which is about}$$

663 miles. Therefore, the correct answer is choice C.

Problem-Solving and Data Analysis Sample Question 4

County Y consists of two districts. One district has an area of 30 square miles and a population density of 370 people per square mile, and the other district has an area of 50 square miles and a population density of 290 people per square mile. What is the population density, in people per square mile, for all of County Y?

Note that this example is in the student-produced response (SPR) format and therefore has no answer choices. You have to generate and enter the response yourself.

The first district has an area of 30 square miles and a population density of 370 people per square mile, so its total population is

$$30 \text{ square miles} \times \frac{370 \text{ people}}{\text{square mile}}, \text{ or } 11,100 \text{ people. The other district has}$$

an area of 50 square miles and a population density of 290 people per square mile, so its total population is $50 \text{ square miles} \times \frac{290 \text{ people}}{\text{square mile}}$,

or 14,500 people. Thus, County Y has a total population of $11,100 + 14,500 = 25,600$ people and a total area of $30 + 50 = 80$ square miles. Therefore, the population density of County Y is $\frac{25,600}{80}$, or 320 people per square mile.

The Problem-Solving and Data Analysis domain also includes questions involving percentages, which are a type of proportion. These questions may involve the concepts of percentage increase and percentage decrease.

Problem-Solving and Data Analysis Sample Question 5

A furniture store buys its furniture from a wholesaler. For a particular style of table, the usual price of the table is 75% more than the cost of the table from the wholesaler. During a sale, the store sells the table for 15% more than the cost from the wholesaler. If the sale price of the table is \$299, what is the usual price for the table?

- A) \$359
- B) \$455
- C) \$479
- D) \$524

The sale price of the table was \$299. This is equal to the cost from the wholesaler plus 15%. Thus, $\$299 = 1.15$ (cost from the wholesaler), and the cost from the wholesaler is $\frac{\$299}{1.15} = \260 . The usual price is the cost from the wholesaler, \$260, plus 75%. Therefore, the usual price the store charges for the table is $1.75 \times \$260 = \455 , which is choice B.

Interpreting Relationships Presented in Scatterplots, Graphs, Tables, and Equations

The behavior of a variable and the relationship between two variables in a context may be explored by considering data presented in scatterplots, tables, and graphs.

The relationship between two quantitative variables may be modeled by a function or an equation. The model may allow very accurate predictions, as, for example, models used in physical sciences, or may only describe a general trend, with considerable variability between the actual and predicted values, as, for example, models used in behavioral and social sciences.

Questions in the Math section may assess your ability to understand and analyze relationships between two variables, the properties of the functions used to model these relationships, and the conditions under which a model is considered to be an appropriate representation of the data. Problem-Solving and Data Analysis questions of these sorts focus on linear, quadratic, and exponential relationships.

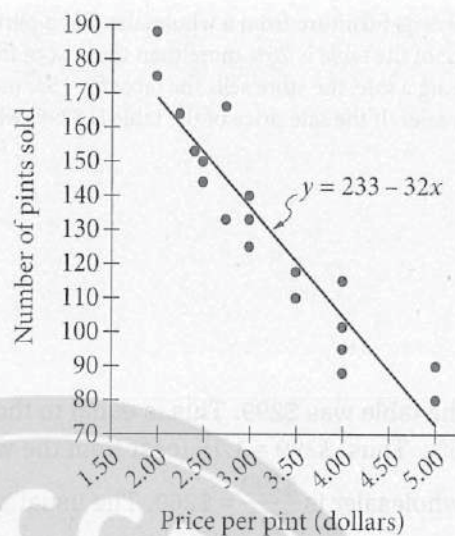
REMINDER

Percent is a type of proportion that means "per 100"; 20%, for instance, means 20 out of (or per) 100. Percent increase or decrease is calculated by finding the difference between two quantities, then dividing the difference by the original quantity and multiplying the result by 100.

REMINDER

The abilities of interpreting and synthesizing data from scatterplots, graphs, and tables are widely applicable in college and in many careers and thus are tested in the Math section.

Problem-Solving and Data Analysis Sample Question 6



A grocery store sells pints of raspberries and sets the price per pint each week. The scatterplot above shows the price and the number of pints of raspberries sold for 19 weeks, along with a line of best fit for the data and an equation for the line of best fit.

Several different questions could be asked about this context.

REMINDER

A line of best fit is a straight line that best represents the association shown on a scatterplot. It's often written in $y = a + bx$ form.

A. According to the line of best fit, how many pints of raspberries would the grocery store be predicted to sell in a week when the price of raspberries is \$4.50 per pint?

Because the line of best fit has equation $y = 233 - 32x$, where x is the price, in dollars, for a pint of raspberries and y is the predicted number of pints of raspberries sold, the number of pints the store would be predicted to sell in a week when the price of raspberries is \$4.50 per pint is $233 - 32(4.50) = 89$ pints.

B. For how many of the 19 weeks shown was the number of pints of raspberries sold greater than the number predicted by the line of best fit?

For a given week, the number of pints of raspberries sold is greater than the number predicted by the line of best fit if and only if the point representing that week lies above the line of best fit. For example, at the price of \$5 per pint, the number sold in two different weeks was approximately 80 and 90, which is more than the 73 predicted by the line of best fit. Of the 19 points, 8 lie above the line of best fit, so there were 8 weeks in which the number of pints sold was greater than what was predicted by the line of best fit.

C. What is the best interpretation of the slope of the line of best fit in this context?

In the Math section, this question would be followed by multiple-choice answer options. The slope of the line of best fit is -32 . This means that the correct answer would state that for each dollar that the price of a pint of raspberries increases, the store is predicted to sell 32 fewer pints of raspberries.

D. What is the best interpretation of the y-intercept of the line of best fit in this context?

In the Math section, this question would be followed by multiple-choice answer options.

In this context, the y-intercept doesn't represent a likely scenario, so it can't be accurately interpreted in this context. According to the model, the y-intercept means that if the store sold raspberries for \$0 per pint—that is, if the store gave raspberries away—233 people would be expected to accept the free raspberries. However, it's not realistic that the store would give away raspberries, and if it did, it's likely that far more people would accept the free raspberries. Also notice that in this case, the leftmost line on the graph is not the y-axis. The lower-left corner shows the x- and y-coordinates of $(1.5, 70)$, not $(0, 0)$.

The fact that the y-intercept indicates that 233 people would accept free raspberries is one limitation of the model. Another limitation is that for a price of \$7.50 per pint or above, the model predicts that a negative number of people would buy raspberries, which is impossible. In general, you should be cautious about applying a model for values outside of the given data. In this example, you should only be confident in the prediction of sales for prices between \$2 and \$5 because those are the lowest and highest prices for which data are shown.

Giving a line of best fit, as in this example, assumes that the relationship between the variables is best modeled by a linear function, but that isn't always true. In the Math section, you may see data that are best modeled by a linear, quadratic, or exponential model.

Problem-Solving and Data Analysis Sample Question 7

Time (hours)	Number of bacteria
0	1,000
1	4,000
2	16,000
3	64,000

The table above gives the initial number (at time $t = 0$) of bacteria placed in a growth medium and the number of bacteria in the growth medium each hour for 3 hours. Which of the following functions best models the number of bacteria, $N(t)$, after t hours?

- A) $N(t) = 4,000t$
- B) $N(t) = 1,000 + 3,000t$
- C) $N(t) = 1,000(4^t)$
- D) $N(t) = 1,000(4^t)$

STRATEGY

To determine whether a relationship is linear or exponential, examine the change in the quantity between successive time periods. If the difference in the quantity is constant, the relationship is linear. If the ratio in the quantity is constant (for instance, 4 times greater than the preceding time period), the relationship is exponential.

The given choices are linear and exponential models. If a quantity is increasing linearly with time, then the *difference* in the quantity between successive time periods is constant. If a quantity is increasing exponentially with time, then the *ratio* in the quantity between successive time periods is constant. According to the table, after each hour, the number of bacteria in the culture is 4 times as great as it was the preceding hour: $\frac{4,000}{1,000} = \frac{16,000}{4,000} = \frac{64,000}{16,000} = 4$. That is, for each increase of 1 in t , the value of $N(t)$ is multiplied by 4. At $t = 0$, which corresponds to the time when the culture was placed in the medium, there were 1,000 bacteria. This is modeled by the exponential function $N(t) = 1,000(4^t)$, which has the value 1,000 at $t = 0$ and increases by a factor of 4 for each increase of 1 in the value of t . Choice D is the correct answer.

The Math section may include questions that require you to know the difference between linear and exponential growth.

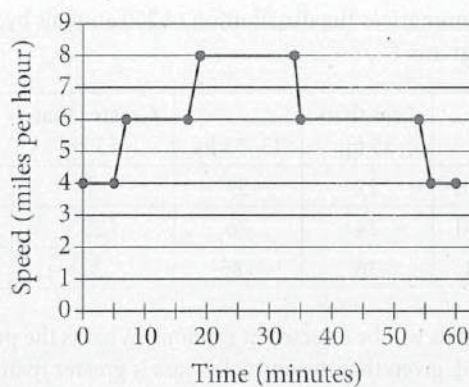
Problem-Solving and Data Analysis Sample Question 8

Every month, Jamal adds two new books to his library. Which of the following types of functions best models the number of books in Jamal's library as a function of time?

- A) Increasing linear
- B) Decreasing linear
- C) Increasing exponential
- D) Decreasing exponential

Over equal intervals, linear functions increase or decrease by a constant amount, while exponential functions increase or decrease by a constant factor. Since the number of books is increasing by a constant amount (2 books) over equal intervals (each month), the function is linear. Also, since the number of books is increasing as time increases, the function is increasing, and therefore choice A is correct.

Problem-Solving and Data Analysis Sample Question 9



One evening, Maria walks, jogs, and runs for a total of 60 minutes. The graph above shows Maria's speed during the 60 minutes. Which segment of the graph represents the times when Maria's speed is the greatest?

- A) The segment from (17, 6) to (19, 8)
- B) The segment from (19, 8) to (34, 8)
- C) The segment from (34, 8) to (35, 6)
- D) The segment from (35, 6) to (54, 6)

The correct answer is choice B. Because the vertical coordinate represents Maria's speed, the part of the graph with the greatest vertical coordinate represents the times when Maria's speed is the greatest. This is the highest part of the graph, the segment from (19, 8) to (34, 8), when Maria runs at 8 miles per hour (mph). Choice A represents the time during which Maria's speed is increasing from 6 to 8 mph; choice C represents the time during which Maria's speed is decreasing from 8 to 6 mph; and choice D represents the longest period of Maria moving at the same speed (6 mph), not the times when Maria's speed is the greatest.

More Data and Statistics

Some questions in the Problem-Solving and Data Analysis domain may assess your ability to understand and analyze data presented in a table, bar graph, histogram, dot plot, box plot, line graph, or other display.

Problem-Solving and Data Analysis Sample Question 10

The given table summarizes the distribution of 200 animals by type of animal and mass, in kilograms.

	Less than 45 kg	45–55 kg	Greater than 55 kg	Total
Cheetah	12	59	4	75
Leopard	24	76	25	125
Total	36	135	29	200

One of these animals will be selected at random. What is the probability of selecting a leopard, given that the animal's mass is greater than 55 kilograms?

- A) $\frac{25}{29}$
B) $\frac{1}{5}$
C) $\frac{29}{200}$
D) $\frac{1}{8}$

The probability of selecting a leopard given that the animal's mass is greater than 55 kg is the number of leopards with a mass greater than 55 kg divided by the total number of animals with a mass greater than 55 kg. According to the table, there are 29 animals with a mass greater than 55 kg. Of these 29 animals, 25 are leopards. Therefore, if the animal has a mass greater than 55 kg, the probability that the animal is a leopard is $\frac{25}{29}$. The correct answer is choice A.

Sample 10 is an example of a conditional probability, the probability of an event given that another event is known to have occurred. The question asks for the probability that an animal chosen at random is a leopard given that the animal has a mass greater than 55 kg.

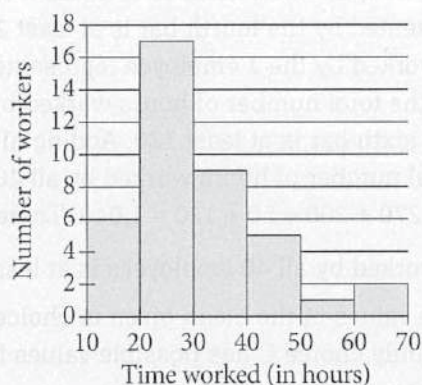
You may be asked to answer questions that involve a measure of center for a data set: the mean or the median. A question may ask you to draw conclusions about one or more of these measures of center even if the exact values cannot be calculated. To recall briefly:

- The *mean* of a set of numerical values is the sum of all the values divided by the number of values in the set.
- The *median* of a set of numerical values is the middle value when the values are listed in increasing (or decreasing) order. If the set has an even number of values, then the median is the average of the two middle values. While technically, the median could be any number between the two middle values, it is most often computed as the average of the two middle values and will always be computed this way for questions in the math section.

REMINDER

Mean and median are measures of center for a data set, while range and standard deviation are measures of spread.

Problem-Solving and Data Analysis Sample Question 11



The histogram shown summarizes the distribution of time worked last week, in hours, by the 40 employees of a landscaping company. In the histogram, the first bar represents all workers who worked at least 10 hours but less than 20 hours; the second bar represents all workers who worked at least 20 hours but less than 30 hours; and so on. Which of the following could be the median and mean amount of time worked, in hours, for the 40 employees?

- A) Median = 22, mean = 23
- B) Median = 24, mean = 22
- C) Median = 26, mean = 32
- D) Median = 32, mean = 30

Note: In the Math section, all questions that include a histogram will include a description of the boundary condition if interpretation of the boundary is required. That is, the question will include a description of how to interpret the values represented by a bar and whether to include the left endpoint but not the right endpoint or the other way around.

If the number of hours the 40 employees worked is listed in increasing order, the median will be a value between the 20th and the 21st numbers in the list. The first 6 numbers in the list will be workers represented by the first bar; hence, each of the first 6 numbers will be at least 10 but less than 20. The next 17 numbers—that is, the 7th through the 23rd numbers in the list—will be workers represented by the second bar; hence, each of the next 17 numbers will be at least 20 but less than 30. Thus, the 20th and the 21st numbers in the list will be at least 20 but less than 30. Therefore, any of the median values in choices A, B, or C is possible, but the median value in choice D isn't.

Now let's find the possible values of the mean. Each of the 6 employees represented by the first bar worked at least 10 hours but less than 20 hours. Thus, the total number of hours worked by these 6 employees is at least 60. Similarly, the total number of hours worked by the 17 employees represented by the second bar is at least 340; the total

REMINDER

The distribution of a variable provides all possible values of the variable and how often they occur.

number of hours worked by the 9 employees represented by the third bar is at least 270; the total number of hours worked by the 5 employees represented by the fourth bar is at least 200; the total number of hours worked by the 1 employee represented by the fifth bar is at least 50; and the total number of hours worked by the 2 employees represented by the sixth bar is at least 120. Adding all these hours shows that the total number of hours worked by all 40 employees is at least $60 + 340 + 270 + 200 + 50 + 120 = 1,040$. Therefore, the mean number of hours worked by all 40 employees is at least $\frac{1,040}{40} = 26$. Therefore, only the values of the mean given in choices C and D are possible. Because only choice C has possible values for both the median and the mean, it's the correct answer.

A data set may have a few values that are much larger or smaller than the rest of the values in the set. These values are called *outliers*. An outlier may represent an important piece of data. For example, if a data set consists of rates of a certain illness in various cities, a data point with a very high value could indicate a serious health issue to be investigated.

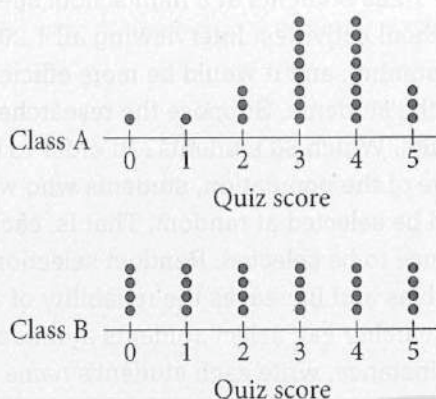
In general, outliers affect the mean more than the median. Therefore, outliers that are larger than the rest of the points in the data set tend to make the mean greater than the median, and outliers that are smaller than the rest of the points in the data set tend to make the mean less than the median. In sample 11, the mean was larger than the median due to the unusually large amount of time worked by a few employees.

REMINDER

You won't be asked to calculate the standard deviation of a set of data in the Math section, but you will be expected to demonstrate an understanding of what standard deviation measures.

The mean and the median are different ways to describe the center of a data set. Another key characteristic of a data set is the amount of variability, or spread, in the data. One measure of spread is the range, which is equal to the maximum value minus the minimum value. Another measure of spread is the standard deviation, which is a measure of how dispersed, or far away, the points in the data set are from the mean value. A low standard deviation indicates the points in the data set are clustered around the mean value, and a high standard deviation indicates the points in the data set are more spread out from the mean value. In the Math section, you won't be asked to compute the standard deviation of a data set, but you do need to understand that a larger standard deviation corresponds to a data set whose values are more spread out from the mean value.

Problem-Solving and Data Analysis Sample Question 12



The dot plots show the distributions of scores on a current events quiz for two classes of 24 students each. Which of the following statements about the standard deviations of the two distributions is true?

- A) The standard deviation of quiz scores in Class A is less than that of quiz scores in Class B.
- B) The standard deviation of quiz scores in Class A is greater than that of quiz scores in Class B.
- C) The standard deviation of quiz scores in Class A is equal to that of quiz scores in Class B.
- D) There is not enough information to compare the standard deviations.

In Class A, the mean score is between 3 and 4. The large majority of scores are 3 and 4, with only a few scores of 0, 1, 2, and 5. In Class B, the mean score is 2.5, and scores are evenly distributed across all possible scores, with many scores not close to the mean score. Because the scores in Class A are more closely clustered around the mean, the standard deviation of the scores in Class A is smaller. The correct answer is choice A.

A *population parameter* is a numerical value that describes a characteristic of a population. For example, the percentage of registered voters who would vote for a certain candidate is a parameter describing the population of registered voters in an election. In another example, the average income of a household in a city is a parameter describing the population of households in that city. We often don't know the value of the population parameter; thus, an essential purpose of statistics is to estimate a population parameter based on a sample from the population. A common example is election polling, where researchers will interview a random sample of registered voters to estimate the proportion of all registered voters who plan to vote for a certain candidate. The precision of the estimate depends on the variability of the sample data and the sample size. For instance, if household incomes in a city vary widely or the sample is small, the estimate that comes from a sample may differ considerably from the actual value for the population parameter.

STRATEGY

When asked to compare the standard deviations of two data sets, first locate the mean approximately. Then ask yourself which data set has values that are more closely clustered around the mean. That data set will have the smaller standard deviation.

For example, a researcher wants to estimate the mean number of hours each week that the 1,200 students at a high school spend on campus engaged in after-school activities. Interviewing all 1,200 students would be time consuming, and it would be more efficient to survey a random sample of the students. Suppose the researcher has time to interview 80 students. Which 80 students? In order to have a sample that's representative of the population, students who will participate in the study should be selected at random. That is, each student must have the same chance to be selected. Random selection is essential in protecting against bias and increases the reliability of any estimates calculated. The researcher can select students at random in several different ways; for instance, write each student's name on a slip of paper, put all the slips in a bowl, mix up the slips, and then draw 80 names from the bowl. In practice, a computer is often used to select participants at random.

If you don't select a random sample, the sampling method used may introduce bias. For example, if you found 80 students from those attending a game of the school's football team, those people would be more likely to be interested in sports and, in turn, an interest in sports might be related to the average amount of time the students spend on the internet. The result would be that the average time those 80 students spend on the internet might not be an accurate estimate of the average amount of time all students at the school spend on the internet.

Suppose you select 80 students at random from the 1,200 students at the high school. You ask them how much time they spend on campus engaged in after-school activities each week, and you calculate that the mean time is 7.9 hours per week. You also find that 6 of the 80 students spend less than 1 hour each week on campus engaged in after-school activities. Based on these results, what conclusions should be made about the entire population of 1,200 students?

Because the sample was selected at random, the mean of 7.9 hours is a plausible estimate of the mean time spent each week on campus engaged in after-school activities by all 1,200 students. Also, we can use the sample data to estimate how many students spend less than 1 hour on campus engaged in after-school activities each week. In the sample, the percentage is $\frac{6}{80}$, or 7.5%. Applying this percentage to the entire population of 1,200 students, the best estimate is that 90 students at the school spend less than 1 hour per week on campus engaged in after-school activities.

REMINDER

You won't need to calculate margins of error in the Math section, but you should understand what the concept means and be able to interpret margins of error in context.

However, the estimates of the population parameters need to be interpreted carefully. An essential part of statistics is accounting for the variability of the estimate. The estimates above are reasonable, but they're unlikely to be exactly correct. Statistical analysis can also describe how far from the estimates the actual values are expected at most to be. To describe the precision of an estimate, statisticians use the concept of *margin of error*. You won't be expected to compute a

margin of error in the Math section, but you should understand how sample size affects margin of error and how to interpret a given margin of error in context.

If the example above had been part of a Math section question, you might have been given survey results indicating that for a random sample of 80 students, the estimated mean was 7.9 hours with an associated margin of error of 0.8 hours. An appropriate interpretation of these results is that it's plausible that the mean number of hours for all 1,200 students in the population is greater than $7.9 - 0.8$, or 7.1 hours, but less than $7.9 + 0.8$, or 8.7 hours. There are two key points to note.

1. The value of the margin of error is affected by two factors: the variability in the data and the sample size. The larger the standard deviation, the larger the margin of error; the smaller the standard deviation, the smaller the margin of error. Furthermore, increasing the size of the random sample provides more information and typically reduces the margin of error.
2. The margin of error applies to the estimated value of the population parameter only; it doesn't inform the estimated value for an individual. In the example, plausible values for the population mean are in the interval from 7.1 hours to 8.7 hours. The time, in hours, that an individual spends on the internet may or may not fall in this interval.

Problem-Solving and Data Analysis Sample Question 13

A quality control researcher at an electronics company is testing the life of the company's batteries in a certain camera. The researcher selects 100 batteries at random from the daily output of the batteries and finds that the life of the batteries has a mean of 342 pictures with an associated margin of error of 18 pictures. Which of the following is the most appropriate conclusion based on these data?

- A) All the batteries produced by the company that day have a life between 324 and 360 pictures.
- B) All the batteries ever produced by the company have a life between 324 and 360 pictures.
- C) It is plausible that the mean life of batteries produced by the company that day is between 324 and 360 pictures.
- D) It is plausible that the mean life of all the batteries ever produced by the company is between 324 and 360 pictures.

The correct answer is choice C. Choices A and B are incorrect because the margin of error gives information about the mean life of all batteries produced by the company that day, not about the life of any individual battery. Choice D is incorrect because the sample of batteries was taken from the population of all the batteries produced by the company on that day. The population of all batteries the company ever produced may have a different mean life because of changes in the formulation of the batteries, wear on machinery, improvements in production processes, and many other factors.

STRATEGY

When a margin of error is provided, determine the value to which the margin of error applies. If the margin of error is associated with a mean, then the margin of error describes the plausible values for individual values in the population.

The statistics examples discussed so far are largely based on investigations intended to estimate some characteristic of a group: the mean amount of time students spend on the internet, the mean life of a battery, and the percentage of registered voters who plan to vote for a candidate. Another primary focus of statistics is to investigate relationships between variables and to draw conclusions about cause and effect. For example, does a new type of physical therapy help people recover from knee surgery faster? For such a study, some people who have had knee surgery will be randomly assigned to the new therapy or to the usual therapy. The medical results of these patients can be compared. The key questions from a statistical viewpoint are

- Is it appropriate to generalize from the sample of patients in the study to the entire population of people who are recovering from knee surgery?
- Is it appropriate to conclude that the new therapy caused any difference in the results for the two groups of patients?

REMINDER

In order for the results of a study to be generalized to an entire population, random sampling is needed. In order for a cause-and-effect relationship to be established, random assignment of individuals to treatments is needed.

The answers depend on the use of random selection and random assignment.

- If the subjects in the sample of a study were selected at random from the entire population in question, the results can be generalized to the entire population because random sampling ensures that each individual has the same chance to be selected for the sample.
- If the subjects in the sample were randomly assigned to treatments, it may be appropriate to make conclusions about cause and effect because the treatment groups will be roughly equivalent at the beginning of the experiment other than the treatment they receive.

This can be summarized in the following table.

	Subjects selected at random	Subjects not selected at random
Subjects randomly assigned to treatments	• Results can be generalized to the entire population. • Conclusions about cause and effect can appropriately be drawn.	• Results <i>cannot</i> be generalized to the entire population. • Conclusions about cause and effect can appropriately be drawn.
Subjects not randomly assigned to treatments	• Results can be generalized to the entire population. • Conclusions about cause and effect should <i>not</i> be drawn.	• Results <i>cannot</i> be generalized to the entire population. • Conclusions about cause and effect should <i>not</i> be drawn.

The previous example discussed treatments in a medical experiment. The word *treatment* refers to any factor that is deliberately varied in an experiment.

Problem-Solving and Data Analysis Sample Question 14

A community center offers a Spanish course. This year, all students in the course were offered additional audio lessons they could take at home. The students who took these additional audio lessons did better in the course than students who didn't take the additional audio lessons. Based on these results, which of the following is the most appropriate conclusion?

- A) Taking additional audio lessons will cause an improvement for any student who takes any foreign language course.
- B) Taking additional audio lessons will cause an improvement for any student who takes a Spanish course.
- C) Taking additional audio lessons was the cause of the improvement for the students at the community center who took the Spanish course.
- D) No conclusion about cause and effect can be made regarding students at the community center who took the additional audio lessons at home and their performance in the Spanish course.

The correct answer is choice D. The better results of these students may have been a result of being more motivated, as shown in their willingness to do extra work, and not the additional audio lessons. Choice A is incorrect because no conclusion about cause and effect is possible without random assignment to treatments and because the sample was only students taking a Spanish course, so no conclusion can be appropriately made about students taking all foreign language courses. Choice B is incorrect because no conclusion about cause and effect is possible without random assignment to treatments and because the students taking a Spanish course at the community center is not a random sample of all students who take a Spanish course. Choice C is incorrect because the students taking the Spanish course at the community center weren't randomly assigned to use the additional audio lessons or to not use the additional audio lessons.

STRATEGY

Be wary of conclusions that claim a cause-and-effect relationship or that generalize a conclusion to a broader population. Before accepting a conclusion, assess whether the subjects were selected at random from the broader population and whether subjects were randomly assigned to treatments.

Summary: Problem-Solving and Data Analysis Questions

Main purpose: To assess your understanding of proportional relationships, percentages, and probability as well as your ability to use data to analyze and solve problems

Proportion of the test section: About 15 percent (approximately five to seven questions)

Question types

- Ratios, rates, proportional relationships, and units
- Percentages
- One-variable data: distributions and measures of center and spread
- Two-variable data: models and scatterplots
- Probability and conditional probability
- Inference from sample statistics and margin of error
- Evaluating statistical claims: observational studies and experiments

CHAPTER 26

Math: Questions— Problem-Solving and Data Analysis Drills

1

Makayla is planning an event in a 5,400-square-foot room. If there should be at least 8 square feet per person, what is the maximum number of people that could attend this event?

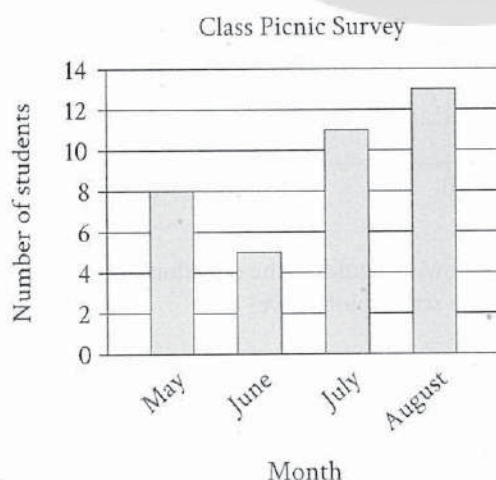
- A) 588
- B) 675
- C) 15,274
- D) 43,200

2

Water flows from a pipe at a rate of 6.0 gallons per minute. How many gallons of water will flow from the pipe in 8.4 minutes?

3

The bar graph shows the results from a survey in which a group of students was asked during which month they prefer to have the class picnic.



Based on the graph, how many students did not respond “May”?

4

A fish hatchery has three tanks for holding fish before they are introduced into the wild. Ten fish weighing less than 5 ounces are placed in tank A. Eleven fish weighing at least 5 ounces but no more than 13 ounces are placed in tank B. Twelve fish weighing more than 13 ounces are placed in tank C. Which of the following could be the median of the weights, in ounces, of these 33 fish?

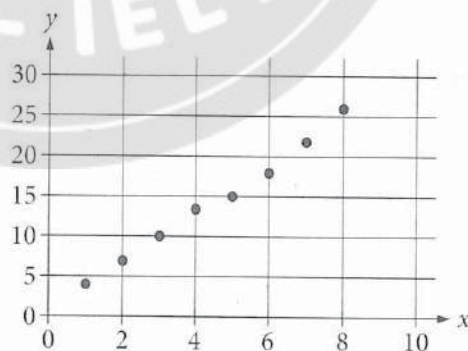
- A) 4.5
- B) 8
- C) 13.5
- D) 15

5

A data set of 27 different numbers has a mean of 33 and a median of 33. A new data set is created by adding 7 to each number in the original data set that is greater than the median and subtracting 7 from each number in the original data set that is less than the median. Which of the following measures does NOT have the same value in both the original and new data sets?

- A) Median
- B) Mean
- C) Sum of the numbers
- D) Standard deviation

6



Which of the following could be the equation for a line of best fit for the data shown in the scatterplot above?

- A) $y = 0.8 + 3x$
- B) $y = 3 + 0.8x$
- C) $y = 3 - 0.8x$
- D) $y = 0.8 - 3x$

7

A sample of 40 fourth-grade students was selected at random from a certain school. The 40 students completed a survey about the morning announcements, and 32 thought the announcements were helpful. Which of the following is the largest population to which the results of the survey can be applied?

- A) The 40 students who were surveyed
- B) All fourth-grade students at the school
- C) All students at the school
- D) All fourth-grade students in the county in which the school is located

8

In which of the following tables is the relationship between the values of x and their corresponding y -values nonlinear?

A)

x	1	2	3	4
y	8	11	14	17

B)

x	1	2	3	4
y	4	8	12	16

C)

x	1	2	3	4
y	8	13	18	23

D)

x	1	2	3	4
y	6	12	24	48

9

A bag containing 10,000 beads of assorted colors is purchased from a craft store. To estimate the percent of red beads in the bag, a sample of beads is selected at random. The percent of red beads in the bag was estimated to be 15%, with an associated margin of error of 2%. If r is the actual number of red beads in the bag, which of the following is most plausible?

- A) $r > 1,700$
- B) $1,300 < r < 1,700$
- C) $200 < r < 1,500$
- D) $r < 1,300$

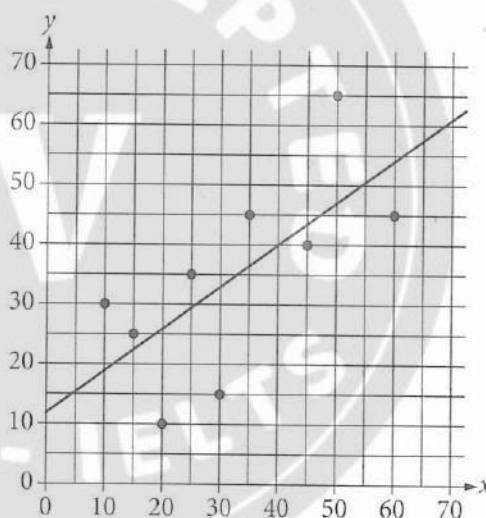
10

The table summarizes the distribution of people in a certain city by age group.

Age group	Percent
Less than 18 years old	27%
18–40 years old	22%
41–65 years old	26%
Greater than 65 years old	25%

If a person in this city is selected at random, what is the probability of selecting a person who is greater than 65 years old, given that the person is at least 18 years old? (Express your answer as a decimal or fraction, not as a percent.)

11



The scatterplot in the xy -plane above shows nine points (x, y) and a line of best fit. Of the following, which best estimates the amount by which the line underestimates the value of y when $x = 50$?

- A) 8
- B) 10
- C) 13
- D) 18

12

Sample	Percent in favor	Margin of error
A	52%	4.2%
B	48%	1.6%

The results of two random samples of votes for a proposition are shown. The samples were selected from the same population, and the margins of error were calculated using the same method. Which of the following is the most appropriate reason that the margin of error for sample A is greater than the margin of error for sample B?

- A) Sample A had a smaller number of votes that could not be recorded.
- B) Sample A had a higher percent of favorable responses.
- C) Sample A had a larger sample size.
- D) Sample A had a smaller sample size.

13

The number of crane flies in a wildlife sanctuary is 30% less than the number of ladybugs in the sanctuary. The number of honeybees in the sanctuary is 85% greater than the number of crane flies in the sanctuary. The number of honeybees in the sanctuary is how many times the number of ladybugs in the sanctuary?

14

The positive number a is 210% of the number b , and a is 30% of the number c . If c is $p\%$ of b , what is the value of p ?

Answer Explanations

QUESTION 1

Choice B is correct. It's given that the event will be in a 5,400-square-foot room and that there should be at least 8 square feet per person. The maximum number of people that could attend the event can be found by dividing the total square feet in the room by the minimum number of square feet needed per person, which gives $\frac{5,400}{8}$, or 675.

Choices A and C are incorrect and may result from conceptual or calculation errors. **Choice D** is incorrect and may result from multiplying, rather than dividing, 5,400 by 8.

QUESTION 2

The correct answer is 50.4. It's given that water flows from a pipe at a rate of 6.0 gallons per minute. It follows that the number of gallons of water that will flow from the pipe in 8.4 minutes can be determined by multiplying the water flow rate by the number of minutes: $\left(\frac{6.0 \text{ gallons}}{1 \text{ minute}}\right)(8.4 \text{ minutes})$, or 50.4 gallons.

Therefore, 50.4 gallons of water will flow from the pipe in 8.4 minutes. Note that 50.4 and $252/5$ are examples of ways to enter a correct answer.

QUESTION 3

The correct answer is 29. It's given that the bar graph shows the results from a survey in which a group of students was asked during which month they prefer to have the class picnic. Based on the graph, 8 students responded "May," 5 students responded "June," 11 students responded "July," and 13 students responded "August." It follows that the number of students who did not respond "May" is the sum of the number of students who responded "June," "July," or "August," which yields $5 + 11 + 13$, or 29. Therefore, 29 students did not respond "May."

QUESTION 4

Choice B is correct. The median of a set of numbers is the middle number when the values in the set are ordered from least to greatest. There are 33 fish, so in an ordered list of the weights, the 17th value would be the median weight. The 10 fish in tank A weigh the least, and these 10 weights would be the first 10 values on the ordered list. The 11 fish in tank B have the next set of greater weights and so would be the 11th through 21st weights in the ordered list, which includes the median weight as the 17th value. The fish in tank B weigh at least 5 ounces but no more than 13 ounces. Of the given choices, only 8 ounces falls within this range of values.

Choice A is incorrect. It's given that tank A has 10 fish weighing less than 5 ounces. Since there are more than 10 fish in tanks B and C combined, the median weight cannot be less than 5 ounces. **Choices C and D** are incorrect. It's given that tank C has 12 fish weighing more than 13 ounces. There are more than 12 fish in tanks A and B combined, so the median weight can't be more than 13 ounces.

QUESTION 5

Choice D is correct. When a data set has an odd number of elements, the median can be found by ordering the values from least to greatest and determining the middle value. Out of the 27 different numbers in this data set, 13 numbers are below the median, one number is exactly 33, and 13 numbers are above the median. When 7 is subtracted from each number below the median and added to each number above the median, the distance between these numbers and the median is larger in the new data set compared to the original data set. Since the median of this data set, 33, is equivalent to the mean of the data set, it follows that the distance between these numbers and the mean is larger in the new data set compared to the original data set. Since standard deviation is a measure of how spread out the data are from the mean, a greater spread from the mean indicates an increased standard deviation. Therefore, the standard deviation doesn't have the same value in both the original and new data sets.

Choice A is incorrect. All the numbers less than the median decrease and all the numbers greater than the median increase, but the median itself doesn't change. **Choices B** and **C** are incorrect. The mean of a data set is found by dividing the sum of the values by the number of values. The net change from subtracting 7 from 13 numbers and adding 7 to 13 numbers is zero. Therefore, neither the mean nor the sum of the numbers changes.

QUESTION 6

Choice A is correct. The data show a strong linear relationship between x and y . The line of best fit for a set of data is a linear equation that minimizes the distances from the data points to the line. An equation for the line of best fit can be written in slope-intercept form, $y = mx + b$, where m is the slope of the graph of the line and b is the y -coordinate of the y -intercept of the graph. Since, for the data shown, the y -values increase as the x -values increase, the slope of a line of best fit must be positive. The data shown lie almost in a line, so the slope can be roughly estimated using the formula for slope, $m = \frac{y_2 - y_1}{x_2 - x_1}$, where (x_1, y_1) and (x_2, y_2) represent two points on the line. The leftmost and rightmost data points have coordinates of about $(1, 4)$ and $(8, 26)$. Substituting $(1, 4)$ for (x_1, y_1) and $(8, 26)$ for (x_2, y_2) in the formula for slope yields $\frac{26 - 4}{8 - 1}$, or $\frac{22}{7}$. Thus, the slope is approximately $\frac{22}{7}$, which is a little greater than 3. Extension of the line to the left would intersect the y -axis at about $(0, 1)$. Only choice A represents a line with a slope close to 3 and a y -intercept close to $(0, 1)$.

Choice B is incorrect and may result from switching the slope and y -intercept. The line with a y -intercept of $(0, 3)$ and a slope of 0.8 is farther from the data points than the line with a slope of 3 and a y -intercept of $(0, 0.8)$. **Choices C** and **D** are incorrect. These equations represent lines with negative slopes, not positive slopes.

QUESTION 7

Choice B is correct. Selecting a sample of a reasonable size at random to use for a survey allows the results from that survey to be applied to the population from which the sample was selected, but not beyond this population. In this case, the population from which the sample was selected is all fourth-grade students at a certain school. Therefore, the results of the survey can be applied to all fourth-grade students at the school.

Choice A is incorrect. The results of the survey can be applied to the 40 students who were surveyed. However, this isn't the largest group to which the results of the survey can be applied. *Choices C* and *D* are incorrect. Since the sample was selected at random from among the fourth-grade students at a certain school, the results of the survey can't be applied to other students at the school or to other fourth-grade students who weren't represented in the survey results. Students in other grades in the school or other fourth-grade students in the county may feel differently about announcements than the fourth-grade students at the school do.

QUESTION 8

Choice D is correct. The relationship between the values of x and their corresponding y -values is nonlinear if the rate of change between these pairs of values isn't constant. The table for choice D gives four pairs of values: (1, 6), (2, 12), (3, 24), and (4, 48). Finding the rate of change, or slope, $\frac{y_2 - y_1}{x_2 - x_1}$, between $(x_1, y_1) = (1, 6)$ and $(x_2, y_2) = (2, 12)$ yields $\frac{12 - 6}{2 - 1}$, or 6. Finding the rate of change between $(x_1, y_1) = (2, 12)$ and $(x_2, y_2) = (3, 24)$ yields $\frac{24 - 12}{3 - 2}$, or 12. Finding the rate of change between $(x_1, y_1) = (3, 24)$ and $(x_2, y_2) = (4, 48)$ yields $\frac{48 - 24}{4 - 3}$, or 24. Since the rate of change isn't constant for these pairs of values, this table shows a nonlinear relationship.

Choices A, B, and C are incorrect. The rate of change between the values of x and their corresponding y -values in each of these tables is constant, with slopes of 3, 4, and 5, respectively. Therefore, each of these tables shows a linear relationship.

QUESTION 9

Choice B is correct. It was estimated that 15% of the beads in the bag are red. Since the bag contains 10,000 beads, it follows that there are an estimated $10,000 \times 0.15 = 1,500$ red beads. It's given that the associated margin of error is 2%, or $10,000 \times 0.02 = 200$ beads. If the estimate is too high, there could plausibly be $1,500 - 200 = 1,300$ red beads. If the estimate is too low, there could plausibly be $1,500 + 200 = 1,700$ red beads. Therefore, it is estimated that there are between 1,300 and 1,700 red beads in the bag. Since it's given that r represents the actual number of red beads in the bag, it follows that $1,300 < r < 1,700$.

Choices A and *D* are incorrect and may result from conceptual or calculation errors. *Choice C* is incorrect because 200 is the associated margin of error for the number of red beads, not the lower bound of the range of red beads.

QUESTION 10

The correct answer is $\frac{25}{73}$. It's given that the table summarizes the distribution of people in a certain city by age group. If a person in this city is selected at random and the person is at least 18 years old, the person is either 18–40 years old, 41–65 years old, or greater than 65 years old. Therefore, the percent of the people in the city who are at least 18 years old is the sum of the percentages of the people who are 18–40 years old, 41–65 years old, and greater than 65 years old, which is $22\% + 26\% + 25\%$, or 73%. It follows that the proportion of people in the city who are at least 18 years old is $\frac{73}{100}$, or 0.73. Based on the table, 25% of the people in the city are greater than 65 years old. It follows that the proportion of people in the city who are greater than 65 years old is $\frac{25}{100}$, or 0.25. Therefore, the probability of selecting a person who is greater

than 65 years old, given that the person is at least 18 years old, is the proportion of people in the city who are greater than 65 years old divided by the proportion of people in the city who are at least 18 years old, or $\frac{0.25}{0.73}$, which is equivalent to $\frac{25}{73}$. Note that $25/73$, $.3424$, $.3425$, and 0.342 are examples of ways to enter a correct answer.

QUESTION 11

Choice D is correct. At $x = 50$, the point on the scatterplot is $(50, 65)$ and the point on the line of best fit is approximately $(50, 47)$. Therefore, the amount by which the line of best fit underestimates the value of y at $x = 50$ is approximately $65 - 47$, or 18.

Choices A, B, and C are incorrect and may result from conceptual or calculation errors.

QUESTION 12

Choice D is correct. In general, a smaller sample size generally leads to a larger margin of error because the sample may be less representative of the whole population.

Choice A is incorrect. The margin of error will depend on the size of the sample of recorded votes, not the number of votes that could not be recorded. In any case, the smaller number of votes that could not be recorded for sample A would tend to decrease, not increase, the comparative size of the margin of error. **Choice B** is incorrect. Since the percent in favor for sample A is the same distance from 50% as the percent in favor for sample B, the percent of favorable responses doesn't affect the comparative size of the margin of error for the two samples. **Choice C** is incorrect and may result from a conceptual error.

QUESTION 13

The correct answer is 1.295. For this wildlife sanctuary, let x be the number of crane flies, y be the number of ladybugs, and z be the number of honeybees. It's given that the number of crane flies in the sanctuary is 30% less than the number of ladybugs. This means that the number of crane flies is $(100 - 30)\%$, or 70%, of the number of ladybugs. The equation $x = \frac{70}{100}y$, or $x = 0.70y$, represents this situation. It's given that the number of honeybees in the sanctuary is 85% greater than the number of crane flies in the sanctuary. This means that the number of honeybees is $(100 + 85)\%$, or 185%, of the number of crane flies. The equation $z = \frac{185}{100}x$, or $z = 1.85x$, represents this situation. Substituting $0.70y$ for x in the equation $z = 1.85x$ yields $z = 1.85(0.70y)$, or $z = 1.295y$. Therefore, the number of honeybees is 1.295 times the number of ladybugs in the sanctuary.

QUESTION 14

The correct answer is 700. It's given that the positive number a is 210% of the number b . Thus, $a = \frac{210}{100}b$. It's also given that a is 30% of the number c . Thus, $a = \frac{30}{100}c$. Since $a = \frac{210}{100}b$ and $a = \frac{30}{100}c$, it follows that $\frac{210}{100}b = \frac{30}{100}c$. Multiplying both sides of this equation by $\frac{100}{30}$ yields $\left(\frac{100}{30}\right)\left(\frac{210}{100}\right)b = c$, or $c = \frac{210}{30}b$. If c is $p\%$ of b , it follows that $\frac{p}{100} = \frac{210}{30}$. Multiplying both sides of this equation by 100 yields $p = \left(\frac{210}{30}\right)(100)$, or $p = 700$.

CHAPTER 27

Math: Questions— Geometry and Trigonometry

In addition to questions in Algebra, Advanced Math, and Problem-Solving and Data Analysis, the Math section includes several questions that are drawn from geometry and trigonometry. They include both multiple-choice and student-produced response questions.

Let's explore the content and skills assessed by these questions.

Geometry

The Math section includes questions that assess your understanding of the key concepts in the geometry of lines, angles, triangles, circles, and other geometric objects. Other questions may also ask you to find the area, surface area, or volume of an abstract figure or a real-life object. You don't need to memorize a large collection of formulas, but you should be comfortable understanding and using these formulas to solve various types of problems. Many of the geometry formulas are provided in the reference sheet available during testing, and less commonly used formulas required to answer particular questions are given in the questions themselves.

To answer geometry questions in the Math section, you should recall the geometry definitions learned before high school and know the essential concepts extended while learning geometry in high school. You should also be familiar with basic geometric notation.

Here are some of the areas that may be the focus of some geometry questions in the Math section.

- Lines and angles
 - ♦ Lengths and midpoints
 - ♦ Measures of angles
 - ♦ Vertical angles
 - ♦ Angle addition
 - ♦ Straight angles and the sum of the angles about a point
 - ♦ Properties of parallel lines and the angles formed when parallel lines are cut by a transversal
 - ♦ Properties of perpendicular lines

REMINDER

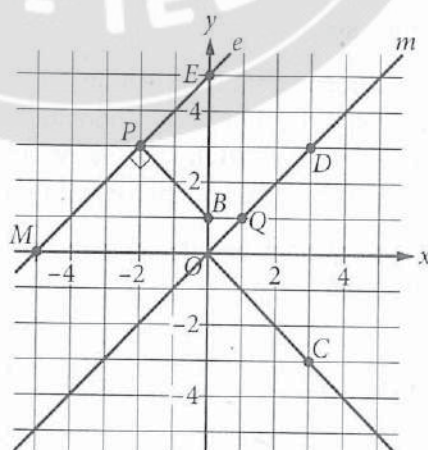
You don't need to memorize a large collection of geometry formulas. Many geometry formulas are provided in the reference sheet available throughout testing.

REMINDER

The triangle inequality theorem states that for any triangle, the length of any side of the triangle must be less than the sum of the lengths of the other two sides of the triangle and greater than the difference of the lengths of the other two sides.

- Triangles and other polygons
 - ♦ Right triangles and the Pythagorean theorem
 - ♦ Properties of equilateral and isosceles triangles
 - ♦ Properties of 30° - 60° - 90° triangles and 45° - 45° - 90° triangles
 - ♦ Congruent triangles and other congruent figures
 - ♦ Similar triangles and other similar figures
 - ♦ The triangle inequality
 - ♦ Squares, rectangles, parallelograms, trapezoids, and other quadrilaterals
 - ♦ Regular polygons
- Circles
 - ♦ Radius, diameter, and circumference
 - ♦ Measure of central angles and inscribed angles
 - ♦ Arc length, arc measure, and area of sectors
 - ♦ Tangents and chords
- Area and volume
 - ♦ Area of plane figures
 - ♦ Volume of solids
 - ♦ Surface area of solids

You should be familiar with the geometric notation for points and lines, line segments, angles and their measures, and lengths.



In the figure shown, the xy -plane has origin O . The values of x on the horizontal x -axis increase as you move to the right, and the values of y on the vertical y -axis increase as you move up. Line e contains point P , which has coordinates $(-2, 3)$; point E , which has coordinates $(0, 5)$; and point M , which has coordinates $(-5, 0)$. Line m passes through the origin $O(0, 0)$, the point $Q(1, 1)$, and the point $D(3, 3)$.

Lines e and m are parallel—they never meet. This is written $e \parallel m$.

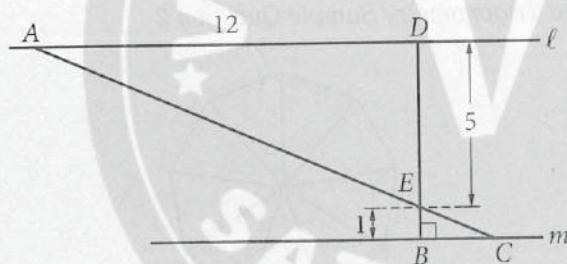
You'll also need to know the following notation:

- $\overleftrightarrow{PE}$: The line containing the points P and E (this is the same as line e)
- $\overline{PE}$, or line segment PE : The line segment with endpoints P and E
- PE : The length of segment PE (you can write $PE = 2\sqrt{2}$)
- $\overrightarrow{PE}$: The ray starting at point P and extending indefinitely in the direction of point E
- $\overrightarrow{EP}$: The ray starting at point E and extending indefinitely in the direction of point P
- $\angle DOC$: The angle formed by $\overrightarrow{OD}$ and $\overrightarrow{OC}$
- $\triangle PEB$: The triangle with vertices P , E , and B
- Quadrilateral $BPMO$: The quadrilateral with vertices B , P , M , and O
- $\overline{BP} \perp \overline{PM}$: Segment BP is perpendicular to segment PM . (You should also recognize that the right-angle box within $\angle BPM$ means this angle is a right angle.)

STRATEGY

Familiarize yourself with these notations in order to avoid confusion on test day.

Geometry and Trigonometry Sample Question 1



In the figure shown, line ℓ is parallel to line m , segment BD is perpendicular to line m , and segment AC and segment BD intersect at E . What is the length of segment AC ?

Since segment AC and segment BD intersect at E , $\angle AED$ and $\angle CEB$ are vertical angles, and so the measure of $\angle AED$ is equal to the measure of $\angle CEB$. Since line ℓ is parallel to line m , $\angle BCE$ and $\angle DAE$ are alternate interior angles of parallel lines cut by a transversal, and so the measure of $\angle BCE$ is equal to the measure of $\angle DAE$. By the angle-angle similarity theorem, $\triangle AED$ is similar to $\triangle CEB$, with vertices A , E , and D corresponding to vertices C , E , and B , respectively.

Also, $\triangle AED$ is a right triangle, so by the Pythagorean theorem, $AE = \sqrt{AD^2 + DE^2}$. Substituting values for the length of the line segments gives $\sqrt{12^2 + 5^2} = \sqrt{169}$, which means that $AE = 13$. Since $\triangle AED$ is similar to $\triangle CEB$, the ratios of the lengths of corresponding sides of the two triangles are in the same proportion. Since $\frac{ED}{EB} = \frac{AE}{EC}$, then $\frac{5}{1} = \frac{13}{EC}$, and so $EC = \frac{13}{5}$. Therefore, $AC = AE + EC$, or $13 + \frac{13}{5} = \frac{78}{5}$.

STRATEGY

A shortcut here is remembering that 5, 12, 13 is a Pythagorean triple (5 and 12 are the lengths of the sides of the right triangle, and 13 is the length of the hypotenuse). Another common Pythagorean triple is 3, 4, 5.

STRATEGY

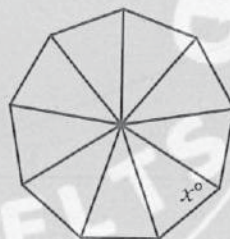
Note how sample 1 requires the knowledge and application of numerous fundamental geometry concepts. Develop mastery of the fundamental concepts, and practice applying them on test-like questions.

Note some of the key concepts that were used in sample 1:

- Vertical angles have the same measure.
- When parallel lines are cut by a transversal, the alternate interior angles have the same measure.
- If two angles of a triangle are congruent to (have the same measure as) two angles of another triangle, the two triangles are similar.
- The Pythagorean theorem: $a^2 + b^2 = c^2$, where a and b are the lengths of the legs of a right triangle and c is the length of the hypotenuse.
- If two triangles are similar, then all ratios of lengths of corresponding sides are equal.
- If point E lies on line segment AC , then $AC = AE + EC$.

Note that if two triangles or other polygons are similar or congruent, the order in which the vertices are named does *not* necessarily indicate how the vertices correspond in the similarity or congruence. Thus, it was stated explicitly in sample 1 that " $\triangle AED$ is similar to $\triangle CEB$, with vertices A , E , and D corresponding to vertices C , E , and B , respectively."

Geometry and Trigonometry Sample Question 2



In the figure shown, a regular polygon with 9 sides has been divided into 9 congruent isosceles triangles by line segments drawn from the center of the polygon to its vertices. What is the value of x ?

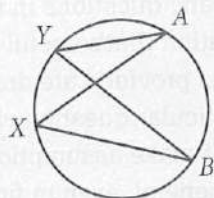
The sum of the measures of the angles around a point is 360° . Since the 9 triangles are congruent, the measures of each of the 9 angles around the center point are equal. Thus, the measure of each of the 9 angles around the center point is $\frac{360^\circ}{9} = 40^\circ$. In any triangle, the sum of the measures of the interior angles is 180° . So in each triangle, the sum of the measures of the remaining two angles is $180^\circ - 40^\circ = 140^\circ$. Since each triangle is isosceles, the measure of each of these two angles is the same. Therefore, the measure of each of these angles is $\frac{140^\circ}{2} = 70^\circ$. Hence, the value of x is 70.

Note some of the key concepts that were used in sample 2:

- The sum of the measures of the angles about a point is 360° .
- Corresponding angles of congruent triangles have the same measure.

- The sum of the measure of the interior angles of any triangle is 180° .
- In an isosceles triangle, the angles opposite the sides of equal length are of equal measure.

Geometry and Trigonometry Sample Question 3



In the figure shown, $\angle AXB$ and $\angle AYB$ are inscribed in the circle. Which of the following statements is true?

- A) The measure of $\angle AXB$ is greater than the measure of $\angle AYB$.
- B) The measure of $\angle AXB$ is less than the measure of $\angle AYB$.
- C) The measure of $\angle AXB$ is equal to the measure of $\angle AYB$.
- D) There is not enough information to determine the relationship between the measure of $\angle AXB$ and the measure of $\angle AYB$.

Choice C is correct. Let the measure of arc AB be d° . Since $\angle AXB$ is inscribed in the circle and intercepts arc AB , the measure of $\angle AXB$ is equal to half the measure of arc AB . Thus, the measure of $\angle AXB$ is $\frac{d^\circ}{2}$. Similarly, since $\angle AYB$ is also inscribed in the circle and intercepts arc AB , the measure of $\angle AYB$ is also $\frac{d^\circ}{2}$. Therefore, the measure of $\angle AXB$ is equal to the measure of $\angle AYB$.

Note the key concept that was used in sample 3:

- The measure of an angle inscribed in a circle is equal to half the measure of its intercepted arc.

You also should know these related concepts:

- The measure of a central angle in a circle is equal to the measure of its intercepted arc.
- An arc is measured in degrees, while arc length is measured in linear units.

You should also be familiar with notation for arcs and circles for questions in the Math section.

- A circle may be identified by the point at its center—for instance, “the circle centered at point M ” or “the circle with center at point M .”
- An arc named with only its two endpoints, such as $\widehat{AB}$ or arc AB , will always refer to a minor arc. A minor arc has a measure that’s less than 180° .

STRATEGY

At first glance, it may appear as though there’s not enough information to determine the relationship between the two angle measures. One key to this question is identifying what’s the same about the two angle measures. In this case, both angles intercept arc AB .

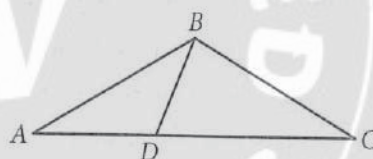
REMINDER

Figures are drawn to scale in the digital SAT Math section unless explicitly stated otherwise. If a question states that a figure isn't drawn to scale, be careful not to make unwarranted assumptions about the figure.

- An arc may also be named with three points: the two endpoints and a third point that the arc passes through. So $\widehat{ACB}$ or arc ACB has endpoints at A and B and passes through point C . Three points may be used to name a minor arc or an arc that has a measure of 180° or more.

In general, figures that accompany questions in the Math section are intended to provide information that's useful in answering the question. Sometimes figures provided are drawn to scale, and sometimes they're not. In a particular question when it's stated that the figure isn't drawn to scale, don't make assumptions about the relative size of angles or segments. In general, even in figures not drawn to scale, the relative positions of points and angles may be assumed to be in the order shown. Also, line segments that extend through points and appear to lie on the same line may be assumed to be on the same line. A point that appears to lie on a line or curve may be assumed to lie on the line or curve.

The text "Note: Figure not drawn to scale" is included with the figure when degree measures may not be accurately shown and specific lengths may not be drawn proportionally. The following example illustrates what information can and can't be assumed from a figure not drawn to scale.



Note: Figure not drawn to scale.

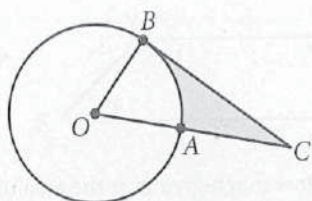
A question may refer to a triangle such as ABC above. Although the note indicates that the figure isn't drawn to scale, you may assume the following from the figure:

- ABD and DBC are triangles.
- A , D , and C are points on a line.
- D is between A and C .
- The length of $\overline{AD}$ is less than the length of $\overline{AC}$.
- The measure of angle ABD is less than the measure of angle ABC .

You may *not* assume the following from the figure:

- The length of $\overline{AD}$ is less than the length of $\overline{DC}$.
- The measures of angles BAD and DBA are equal.
- The measure of angle DBC is greater than the measure of angle ABD .
- Angle DBC is a right angle.

Geometry and Trigonometry Sample Question 4



In the given figure, O is the center of the circle, segment BC is tangent to the circle at B , and A lies on segment OC . If the lengths of $\overline{OB}$ and $\overline{AC}$ are each 6, what is the area of the shaded region?

- A) $18\sqrt{3} - 3\pi$
- B) $18\sqrt{3} - 6\pi$
- C) $36\sqrt{3} - 3\pi$
- D) $36\sqrt{3} - 6\pi$

Since line segment BC is tangent to the circle at B , it follows that $\overline{BC} \perp \overline{OB}$, and so triangle OBC is a right triangle with its right angle at B . Since $OB = 6$ and $\overline{OB}$ and $\overline{OA}$ are both radii of the circle, $OA = 6$. Since $OC = OA + AC$, $OC = 6 + 6$, or 12. Thus, triangle OBC is a right triangle with the length of the hypotenuse ($OC = 12$) twice the length of one of its legs ($OB = 6$). It follows that triangle OBC is a 30° - 60° - 90° triangle with its 30° angle at C and its 60° angle at O . The area of the shaded region is the area of triangle OBC minus the area of the sector bounded by radii OA and OB .

In the 30° - 60° - 90° triangle OBC , the length of side OB , which is opposite the 30° angle, is 6. Thus, the length of side BC , which is opposite the 60° angle, is $6\sqrt{3}$. Hence, the area of triangle OBC is $\frac{1}{2}(6)(6\sqrt{3}) = 18\sqrt{3}$. Since the sector bounded by radii OA and OB has central angle 60° , the area of this sector is $\frac{60}{360} = \frac{1}{6}$ of the area of the circle. Since the circle has radius 6, its area is $\pi(6)^2 = 36\pi$, and so the area of the sector is $\frac{1}{6}(36\pi) = 6\pi$. Therefore, the area of the shaded region is $18\sqrt{3} - 6\pi$, which is choice B.

Note some of the key concepts that were used in sample 4:

- A tangent to a circle is perpendicular to the radius of the circle drawn to the point of tangency.
- Properties of 30° - 60° - 90° triangles.
- Area of a circle.
- The area of a sector with central angle x° is equal to $\frac{x}{360}$ of the area of the entire circle.

STRATEGY

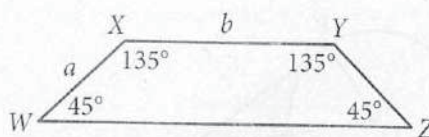
On complex multistep questions such as sample 4, start by identifying the task (finding the area of the shaded region) and considering the intermediate steps that you'll need to solve for (the area of triangle OBC and the area of sector OBA) in order to get to the final answer. Breaking up this question into a series of smaller questions will make it more manageable.

REMINDER

Arc length, area of a sector, and central angle are all proportional to each other in a circle. This proportionality is written as

$$\frac{\text{arc length}}{\text{circumference}} = \frac{\text{central angle}}{360 \text{ degrees}} = \frac{\text{area of a sector}}{\text{area of a circle}}$$

Geometry and Trigonometry Sample Question 5



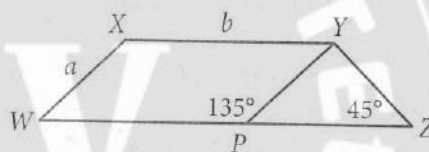
Trapezoid WXYZ is shown. How much greater is the area of this trapezoid than the area of a parallelogram with side lengths a and b and base angles of measures 45° and 135° ?

- A) $\frac{1}{2}a^2$
- B) $\sqrt{2}a^2$
- C) $\frac{1}{2}ab$
- D) $\sqrt{2}ab$

STRATEGY

Note how drawing the parallelogram within trapezoid WXYZ makes it much easier to compare the areas of the two shapes, minimizing the amount of calculation needed to arrive at the solution. Be on the lookout for time-saving shortcuts such as this one.

In the figure, draw a line segment from Y to the point P on side WZ of the trapezoid such that $\angle YPW$ has measure 135° , as shown in the figure below.



Since in trapezoid WXYZ side XY is parallel to side WZ , it follows that $WXYP$ is a parallelogram with side lengths a and b and base angles of measure 45° and 135° . Thus, the area of the trapezoid is greater than the area of a parallelogram with side lengths a and b and base angles of measure 45° and 135° by the area of triangle PYZ . Since $\angle YPW$ has measure 135° , it follows that $\angle YPZ$ has measure 45° . Hence, triangle PYZ is a 45° - 45° - 90° triangle with legs of length a . Therefore, its area is $\frac{1}{2}a^2$, which is choice A.

Note some of the key concepts that were used in sample 5:

- Properties of trapezoids and parallelograms.
- Area of a 45° - 45° - 90° triangle.

Some questions in the Math section may ask you to find the area, surface area, or volume of an object, possibly in context.

Geometry and Trigonometry Sample Question 6

A laboratory supply company produces graduated cylinders, each with an internal radius of 2 inches and an internal height between 7.75 inches and 8 inches. What is one possible volume, rounded to the nearest cubic inch, of a graduated cylinder produced by this company?

The volume of a cylinder can be found by using the formula $V = \pi r^2 h$, where r is the radius of the circular base and h is the height of the cylinder. The smallest possible volume, in cubic inches, of a graduated cylinder produced by the laboratory supply company can be found by substituting 2 for r and 7.75 for h , giving $V = \pi(2^2)(7.75)$. This gives a volume of approximately 97.39 cubic inches, which rounds to 97 cubic inches. The largest possible volume, in cubic inches, can be found by substituting 2 for r and 8 for h , giving $V = \pi(2^2)(8)$. This gives a volume of approximately 100.53 cubic inches, which rounds to 101 cubic inches. Therefore, the possible volumes are all the integers greater than or equal to 97 and less than or equal to 101, which are 97, 98, 99, 100, and 101. Any of these numbers may be entered as the correct answer.

Coordinate Geometry

Questions in the Geometry and Trigonometry domain may ask you to use the coordinate plane and equations of lines and circles to describe figures. You may be asked to create the equation of a circle given the figure or use the structure of a given equation to determine a property of a figure in the coordinate plane. You should know that the graph of $(x - a)^2 + (y - b)^2 = r^2$ in the xy -plane is a circle with center (a, b) and radius r .

Geometry and Trigonometry Sample Question 7

$$x^2 + (y + 1)^2 = 4$$

The graph of the given equation in the xy -plane is a circle. A new circle is drawn whose center is 1 unit up from this circle and whose radius is 1 more than this circle. Which of the following is an equation of this new circle?

- A) $x^2 + y^2 = 5$
- B) $x^2 + y^2 = 9$
- C) $x^2 + (y + 2)^2 = 5$
- D) $x^2 + (y + 2)^2 = 9$

The graph of the given equation $x^2 + (y + 1)^2 = 4$ in the xy -plane is a circle with center $(0, -1)$ and radius $\sqrt{4} = 2$. If the center of the new circle is 1 unit up from the center of the given circle, the center of the new circle will be $(0, 0)$. If the radius of the new circle is 1 more than the given circle, the radius of the new circle will be 3. Therefore, an equation of the new circle in the xy -plane is $x^2 + y^2 = 3^2 = 9$, so choice B is correct.

Geometry and Trigonometry Sample Question 8

$$x^2 + 8x + y^2 - 6y = 24$$

The graph of the given equation in the xy -plane is a circle. What is the radius of the circle?

REMINDER

You should know that the graph of $(x - a)^2 + (y - b)^2 = r^2$ in the xy -plane is a circle with center (a, b) and radius r . You should also be comfortable finding the center or radius of a circle from an equation not written in "standard form" by using the method of completing the square to rewrite the equation in standard form.

The given equation isn't in the standard form $(x - a)^2 + (y - b)^2 = r^2$. You can put it in standard form by completing the square. Since the coefficient of x is 8 and the coefficient of y is -6 , you can write the equation in terms of $(x + 4)^2$ and $(y - 3)^2$ as follows:

$$x^2 + 8x + y^2 - 6y = 24$$

$$(x^2 + 8x + 16) - 16 + (y^2 - 6y + 9) - 9 = 24$$

$$(x + 4)^2 - 16 + (y - 3)^2 - 9 = 24$$

$$(x + 4)^2 + (y - 3)^2 = 24 + 16 + 9$$

$$(x + 4)^2 + (y - 3)^2 = 49$$

Since $49 = 7^2$, the radius of the circle is 7. (Also, the center of the circle is $(-4, 3)$.)

Trigonometry and Radians

Questions in the Geometry and Trigonometry domain may ask you to apply the definitions from right triangle trigonometry. You should also know the definition of radian measure; you may also need to convert between angle measure in degrees and radians. You may need to evaluate trigonometric functions at benchmark angle measures such as 0 , $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$, and $\frac{\pi}{2}$ radians (which are equal to the angle measures 0° , 30° , 45° , 60° , and 90° , respectively). You will *not* be asked for values of trigonometric functions that require a calculator.

For an acute angle, the trigonometric functions sine, cosine, and tangent can be defined using right triangles. (Note the functions are often abbreviated as $\sin$, $\cos$, and $\tan$, respectively.)



For $\angle C$ in the right triangle shown:

$$\sin \angle C = \frac{AB}{BC} = \frac{\text{length of leg opposite } \angle C}{\text{length of hypotenuse}}$$

$$\cos \angle C = \frac{AC}{BC} = \frac{\text{length of leg adjacent to } \angle C}{\text{length of hypotenuse}}$$

$$\tan \angle C = \frac{AB}{AC} = \frac{\text{length of leg opposite } \angle C}{\text{length of leg adjacent to } \angle C} = \frac{\sin \angle C}{\cos \angle C}$$

REMINDER

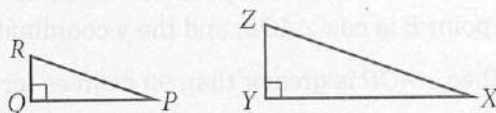
The acronym "SOHCAHTOA" may help you remember how to compute sine, cosine, and tangent. SOH stands for Sine equals Opposite over Hypotenuse, CAH stands for Cosine equals Adjacent over Hypotenuse, and TOA stands for Tangent equals Opposite over Adjacent.

The functions will often be written as $\sin C$, $\cos C$, and $\tan C$, respectively.

Note that the trigonometric functions are actually functions of the *measures* of an angle, not the angle itself. Thus, if the measure of $\angle C$ is, say, 30° , you can write $\sin 30^\circ$, $\cos 30^\circ$, and $\tan 30^\circ$, respectively.

Also note that sine and cosine are cofunctions and that $\sin B = \frac{\text{length of leg opposite } \angle B}{\text{length of hypotenuse}} = \frac{AC}{BC} = \cos C$. This is the complementary angle relationship: $\sin x^\circ = \cos (90^\circ - x^\circ)$.

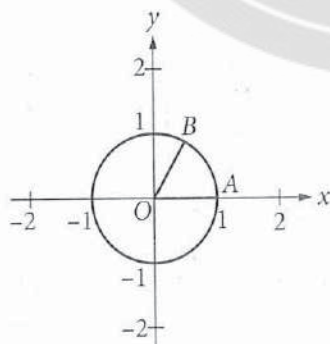
Geometry and Trigonometry Sample Question 9



In the figure, right triangle PQR is similar to right triangle XYZ , with vertices P , Q , and R corresponding to vertices X , Y , and Z , respectively. If $\cos R = 0.263$, what is the value of $\cos Z$?

By the definition of cosine, $\cos R = \frac{RQ}{RP}$ and $\cos Z = \frac{ZY}{ZX}$. Since triangle PQR is similar to triangle XYZ , with vertices P , Q , and R corresponding to vertices X , Y , and Z , respectively, the ratios $\frac{RQ}{RP}$ and $\frac{ZY}{ZX}$ are equal. Therefore, since $\cos R = \frac{RQ}{RP}$, or $\cos R = 0.263$, it follows that $\cos Z = \frac{ZY}{ZX}$, or $\cos Z = 0.263$. Note that to find the values of the trigonometric functions of d° , you can use *any* right triangle with an acute angle of measure d° and then find the appropriate ratio of lengths of sides.

Note that since an acute angle of a right triangle has a measure between 0° and 90° , exclusive, right triangles can be used only to find values of trigonometric functions for angles with measures between 0° and 90° , exclusive. The definitions of sine, cosine, and tangent can be extended to all values. This is done using radian measure and the unit circle.



The circle above has radius 1 and is centered on the origin, O . An angle in the coordinate plane is said to be in *standard position* if it meets these two conditions: (1) its vertex lies at the origin and (2) one of its sides lies along the positive x -axis. Since $\angle AOB$, above, formed by segments OA and OB , meets both these conditions, it's in